# Sistemas de control reseteado: fundamentos y aplicaciones

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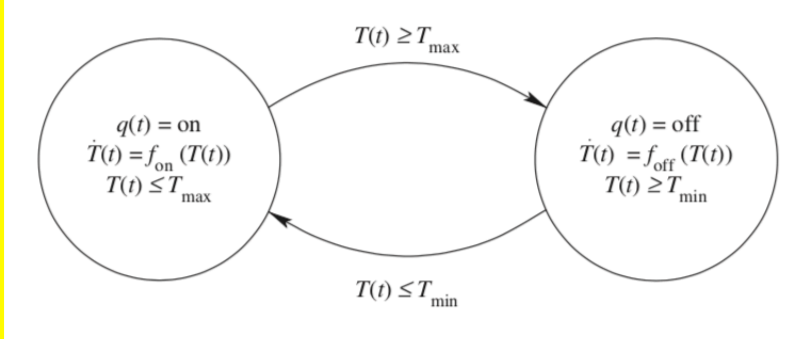
## Un ejemplo sencillo: termostato

Un autómata híbrido



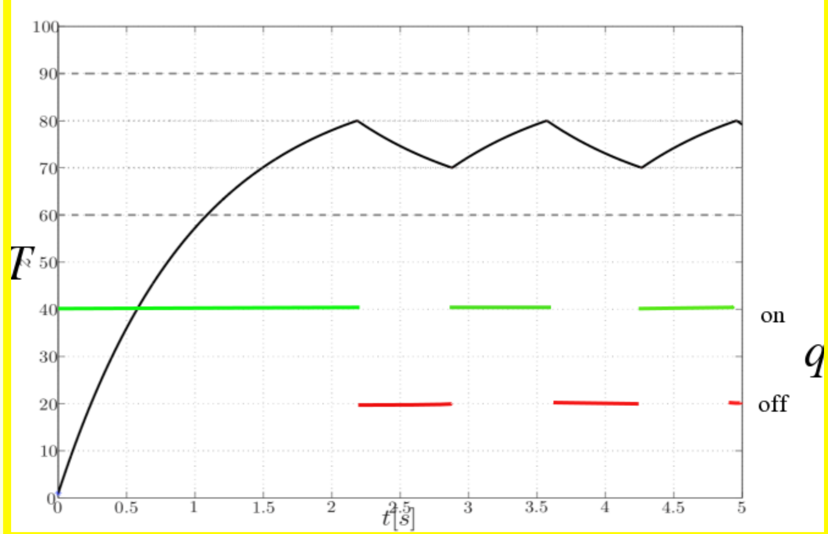

Estado (discreto/lógico): $q \in \{\text{on}, \text{off}\}$

Estado (tiempo continuo): $T \in \text{Reals}$

Eventos: $q = on$ AND $T \leq T_{max}, \cdots$

Dinámica temporal: $f_q \in \{f_{\text{on}}, f_{\text{off}}\}$

Dinámica de eventos discretos: $q = on \longleftrightarrow q = off$



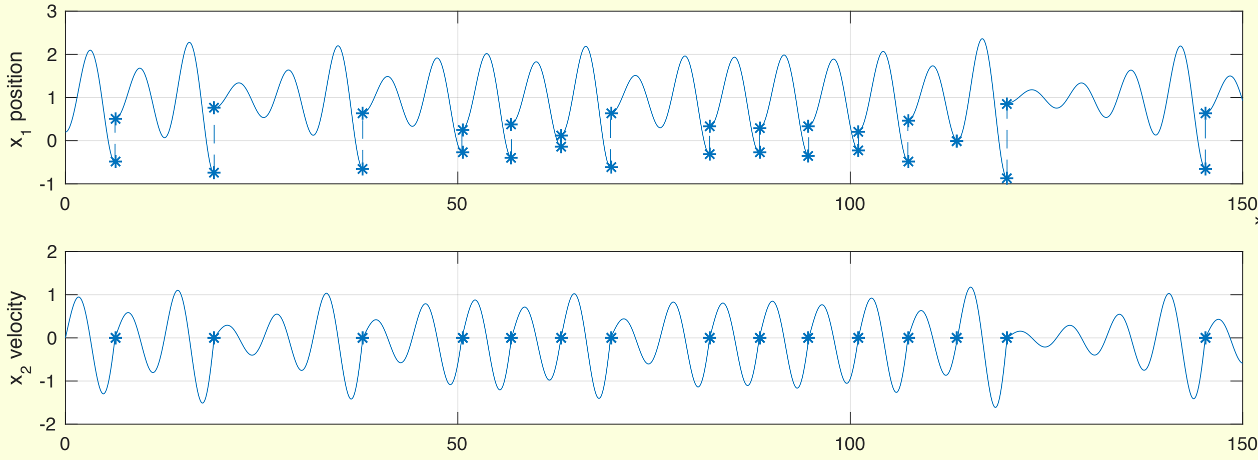

Dinámica de eventos discretos (“switching”) + Dinámica temporal : Sistema híbrido

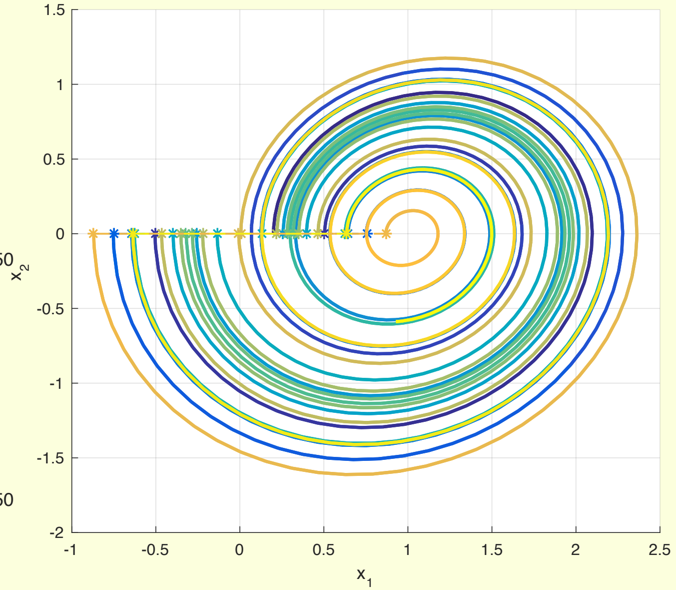
## Otro ejemplo: Un oscilador reseteado

Un sistema dinámico impulsivo:

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 2\delta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} & , \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \notin \mathcal{S} \\ \begin{pmatrix} x_1^+ \\ x_2^+ \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & , \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathcal{S} \end{cases}$$

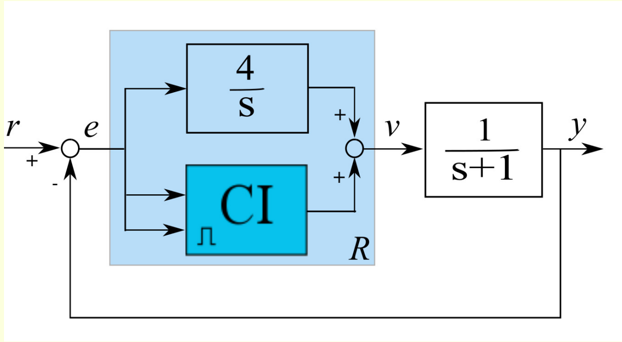
Conjunto de reseteo (eventos): $\mathcal{S} = \{x \in \mathbb{R}^2 : x_1 < 0, x_2 = 0\}$




Dinámica de eventos discretos (reseteo) + Dinámica temporal : Sistema híbrido

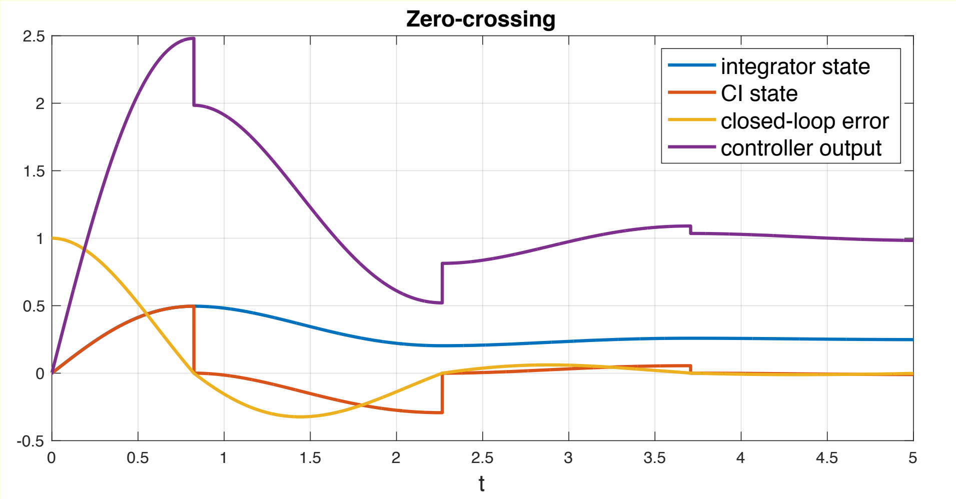
## Otro ejemplo: Controlador reseteado (Controlador de Horowitz)

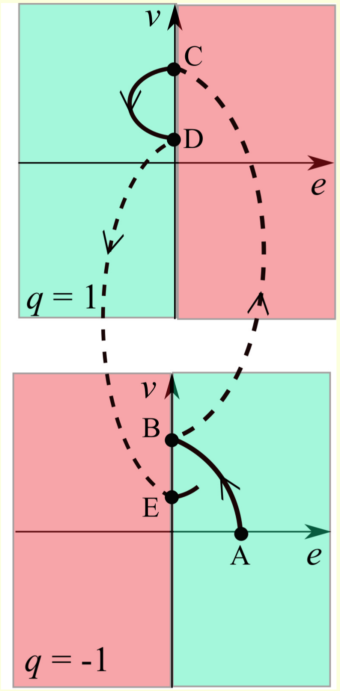



$$
\mathrm{CI}: \begin{cases} \dot{x}_r = e_{CI} & , (x_r, q, e_{CI}, e) \in \mathcal{C} \\ \begin{pmatrix} x_r^+ \\ q^+ \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_r \\ q \end{pmatrix} & , (x_r, q, e_{CI}, e) \in \mathcal{D} \end{cases}
$$

$$
\mathcal{C} = \{(x_r, q, e_{CI}, e) \in \mathcal{O} \times \mathbb{R}^2 : qe \leq 0\}
$$

$$
\mathcal{D} = \{(x_r, q, e_{CI}, e) \in \mathcal{O} \times \mathbb{R}^2 : qe \geq 0\}
$$




Dinámica de eventos discretos (reseteo) + Dinámica temporal : Sistema híbrido

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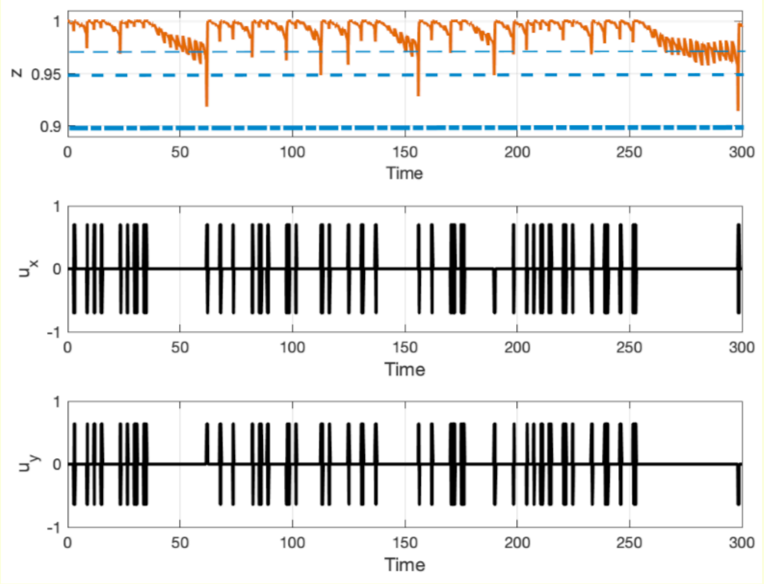
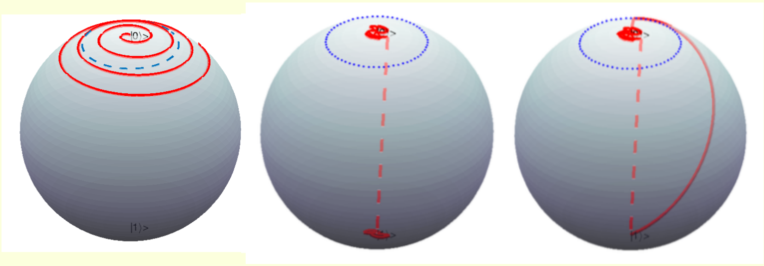
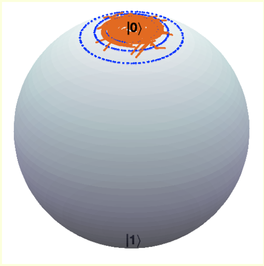
## Otro ejemplo: Control de un “quantum bit” (qubit)

$$
\begin{cases}
\begin{pmatrix} \dot{\mathbf{r}} \\ \dot{T} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} (A + \mathbf{B}\mathbf{u}_q)\mathbf{r} \\ -1 \\ 0 \end{pmatrix} & ,(\mathbf{r}, T, q) \in C \\
\begin{pmatrix} \mathbf{r}^+ \\ T^+ \\ q^+ \end{pmatrix} \in \begin{cases} \begin{pmatrix} R_1(\mathbf{r}) \\ T_1 \\ 1 \end{pmatrix} , v \in [0, \frac{1+\epsilon \mathbf{r}\cdot\mathbf{n}}{2}] \\ \begin{pmatrix} R_{-1}(\mathbf{r}) \\ T_{-1} \\ -1 \end{pmatrix} , v \in [\frac{1+\epsilon \mathbf{r}\cdot\mathbf{n}}{2}, 1] \end{cases} & ,(\mathbf{r}, T, q) \in D \\
v \sim \mu(\cdot)
\end{cases}
$$

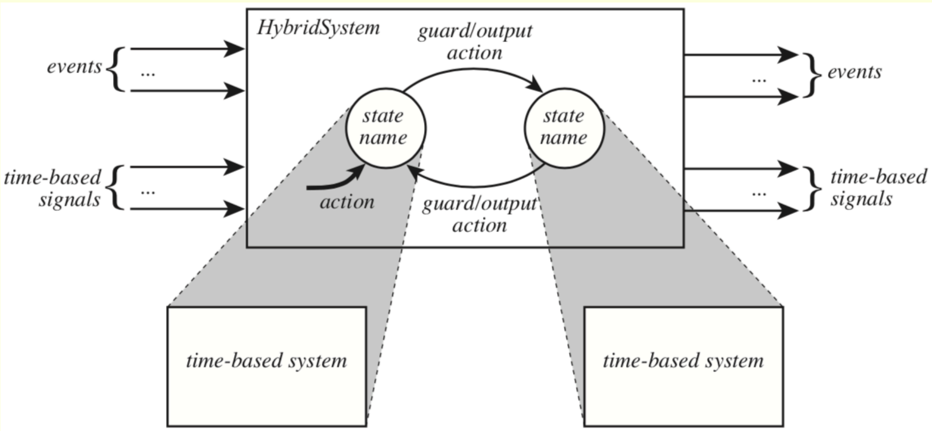


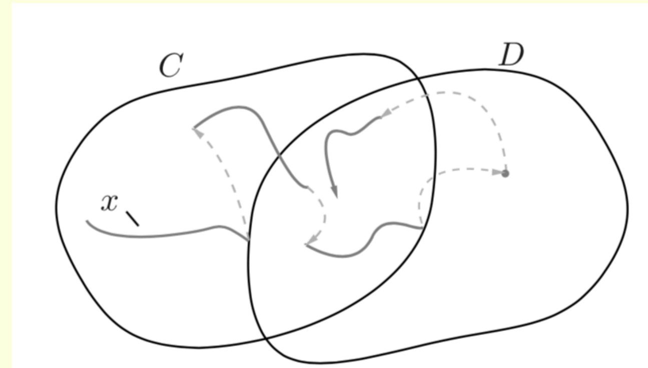

Dinámica de eventos discretos (reseteo) + Dinámica temporal : Sistema híbrido

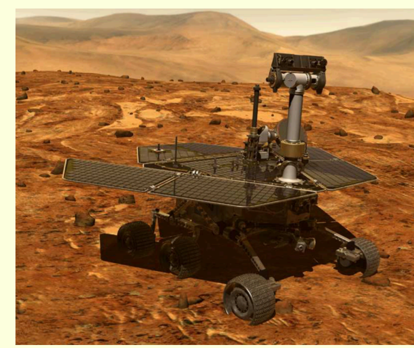
## Hybrid automata




## Hybrid inclusions

$$\begin{cases} \dot{x} \in F(x) & , x \in C \\ x^+ \in G(x) & , x \in D \end{cases}$$



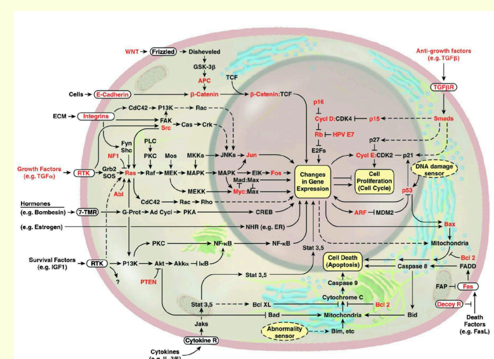



## Hybrid behaviour:

- switching of the (time-driven) dynamics
- state jumps
- events may depends on time/state/external inputs

## Applications in science and engineering are ubiquitous

- Embedded systems
- Computation, communication and control
- Biological systems
- Ciberphysical Systems
- Quantum systems
- ...



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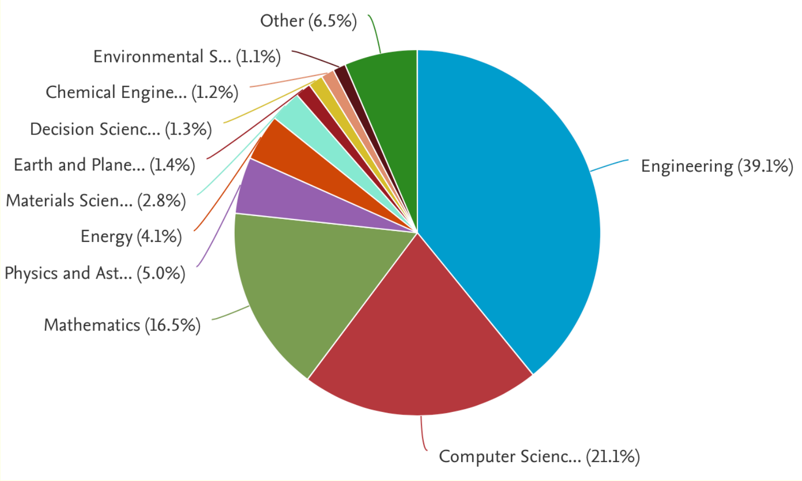
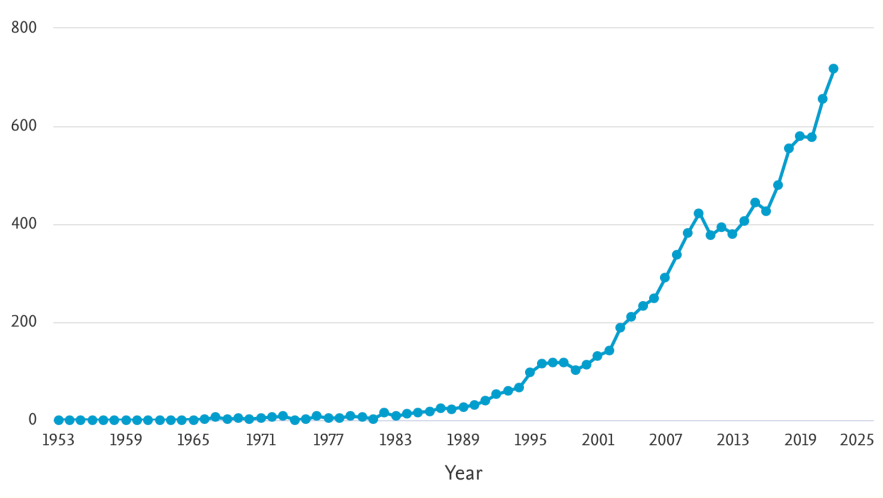
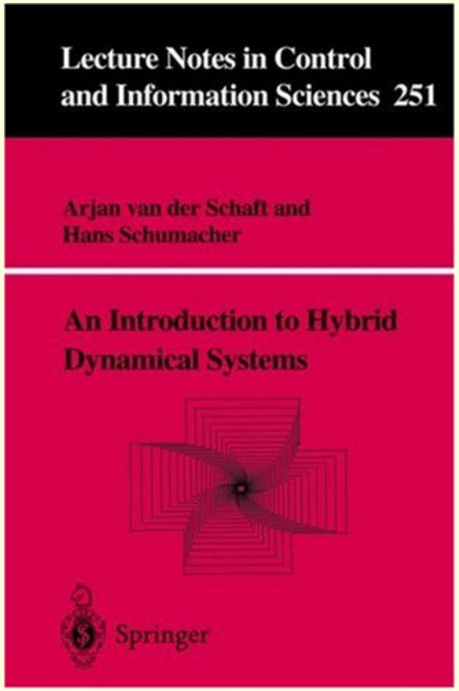
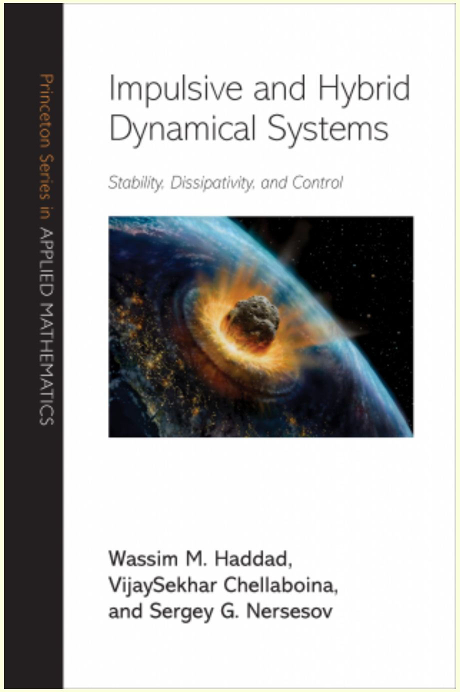
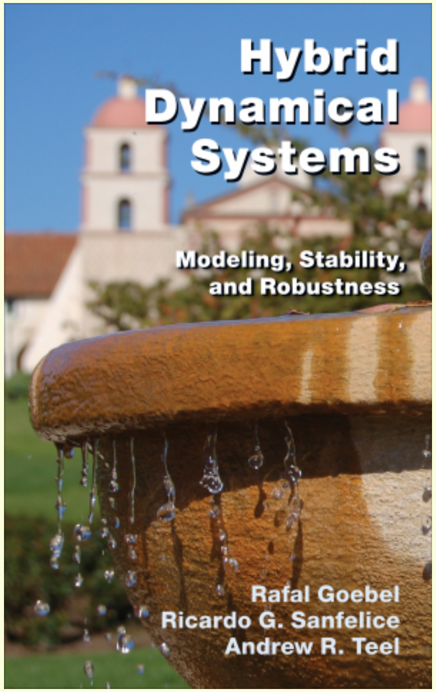
*Fuente*: Scopus, TITLE-ABS-KEY ( "hybrid control" OR "reset control" OR "impulsive control" )



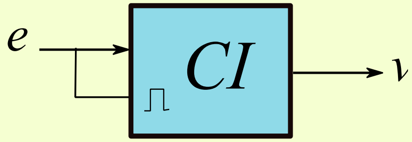
# The Clegg integrator (1958)

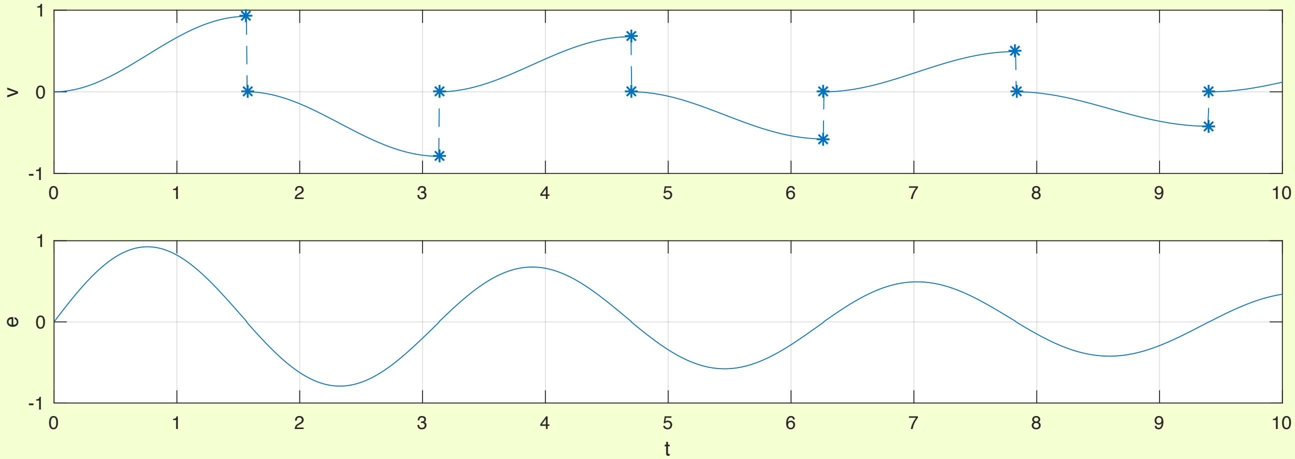
"A nonlinear integrator for servomechanisms"

- **Basic idea**: the integrator state/output is set to zero (reset) at those instants in which the integrator input is zero.

- CI as a hybrid system:

$$\begin{cases} \dot{v}(t) = e(t) & , e \neq 0 \lor v = 0 \\ v(t^+) = 0 & , e = 0 \land v \neq 0 \end{cases}$$




- Inputs must be continuous signals with isolated zeros (e.g. Bohl functions)

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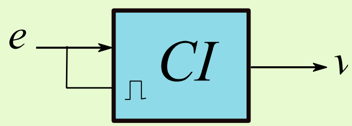
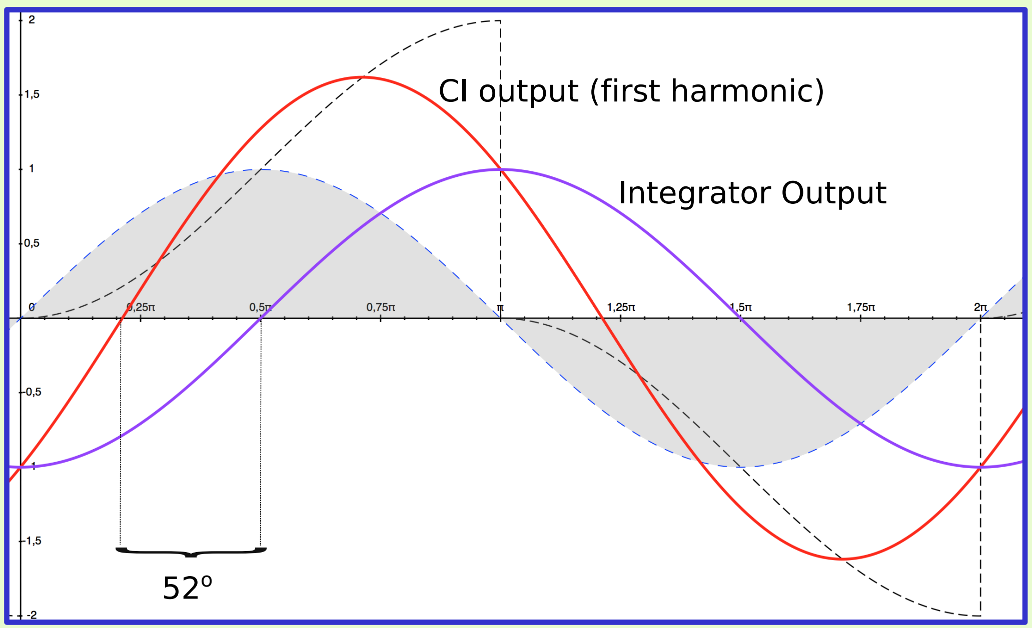
# The Clegg integrator (1958)

"A nonlinear integrator for servomechanisms"

- CI gives extra phase lead in comparison to an (linear) integrator.




$$e(t) = Asin(\omega t) \rightarrow v(t) = \frac{1.6}{\omega} Asin(\omega t - 38.1^\circ) + \cdots$$




$$CI(j\omega) = \frac{1}{j\omega}(1 + j\frac{4}{\pi}) \approx \frac{1.62}{\omega} e^{-j38.1^\circ} \qquad I(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega} e^{-j90^\circ}$$

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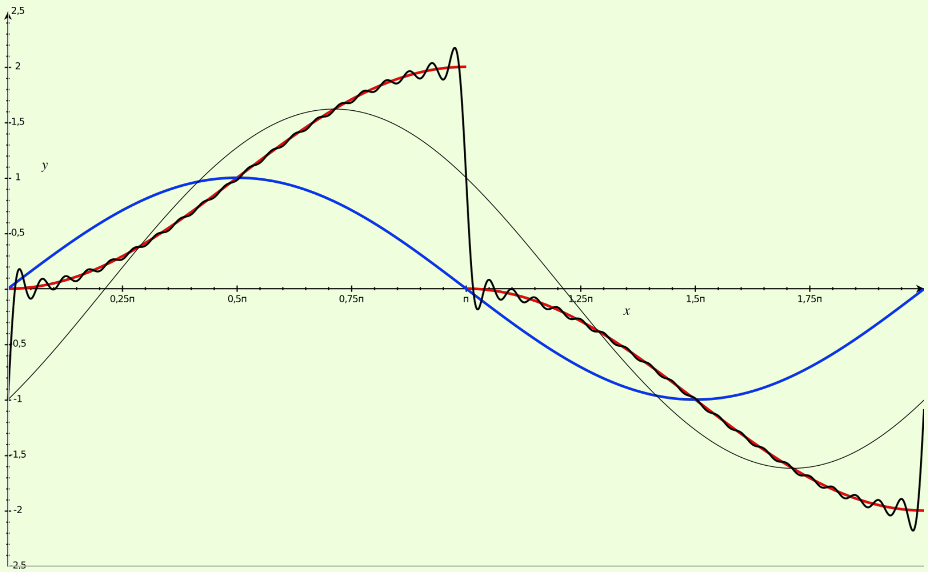
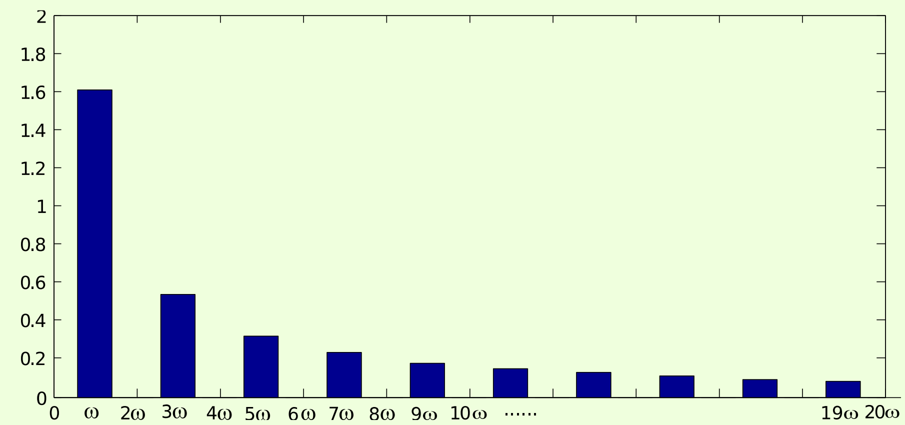
# The Clegg integrator (1958)

“A nonlinear integrator for servomechanisms”

$$u(t) = sin(\omega t) \rightarrow y(t) = \frac{1.62}{\omega}\sin(\omega t - 38.1^\circ) + \frac{0.54}{\omega}\sin(3\omega t - \phi_3) + \frac{0.32}{\omega}\sin(5\omega t - \phi_5) + \cdots$$





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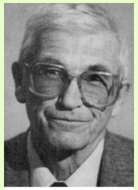
# Horowitz's FORE (1975)



Isaac M. Horowitz (1920–2005)

- FORE gives phase lead over a (linear) first order controller:

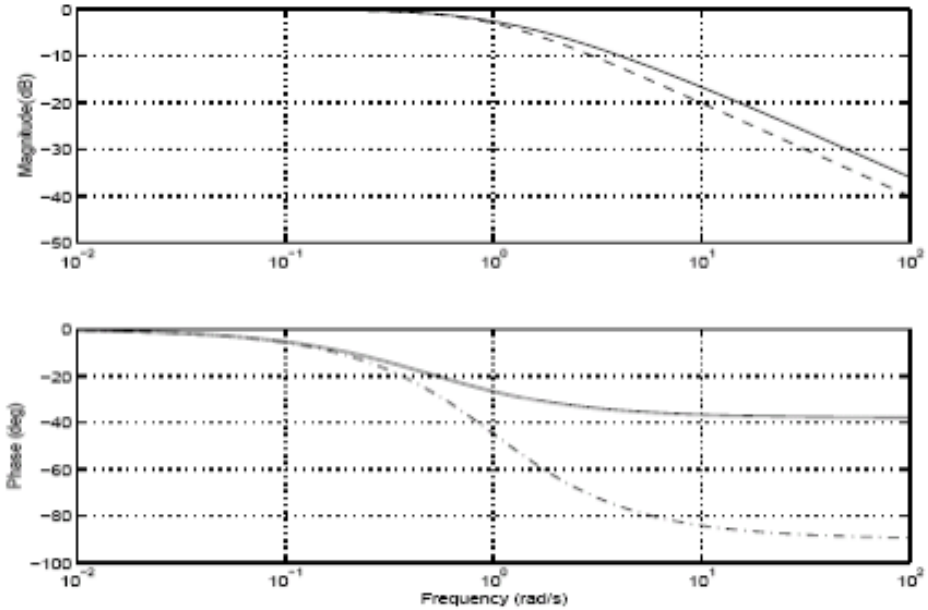
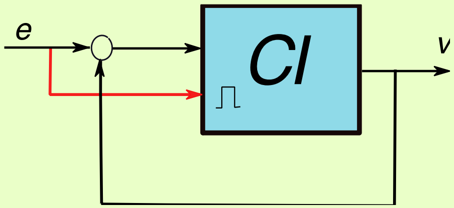



Fig. 4. Bode Diagram of FORE DF (solid line) and its LTI base system (dashed line)

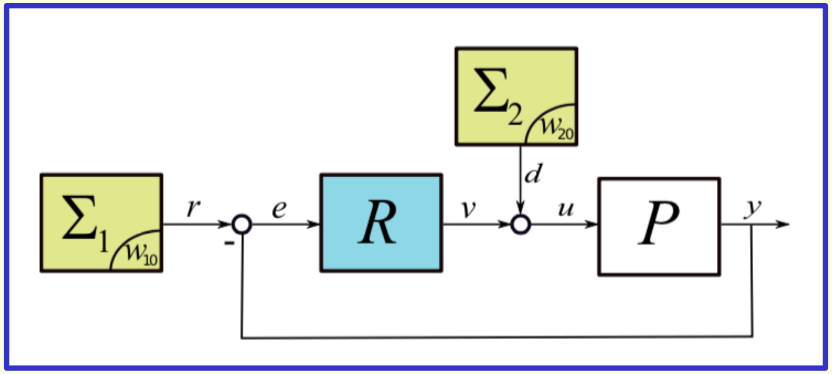



$$\begin{cases} \dot{v}(t) = -av(t) + Ke(t), & e(t) \neq 0 \\ v(t^+) = 0, & e(t) = 0 \end{cases}$$

$$FORE(\omega) = \frac{K}{a + j\omega}\left(1 + j\frac{2}{\pi}\frac{\omega^2}{a^2+\omega^2}(1 + e^{-a\frac{\pi}{\omega}})\right)$$

## Main motivation:
## Overcoming Fundamental Limitations of LTI controllers

- **Basic Idea**: It is NOT possible to satisfy arbitrary design specifications with a feedback LTI controller, even with ideal plants (without uncertainty and without actuators limitations)




- **Basic design specifications:**
  - Well-posedness
  - Stability
  - Disturbance rejection
  - Reference tracking
  - Robustness

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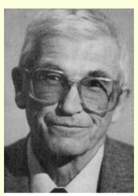
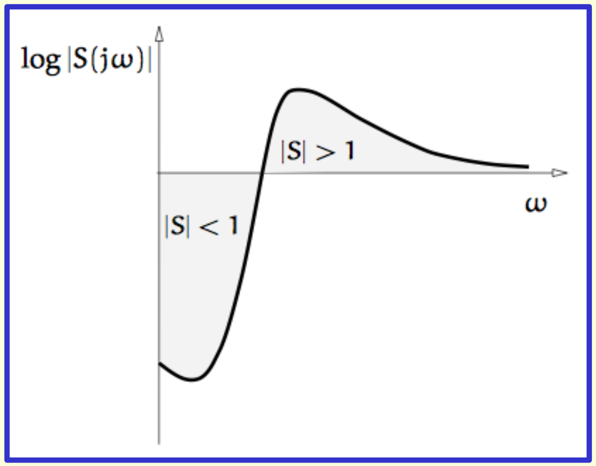
# Main motivation:
# Overcoming Fundamental Limitations of LTI controllers

Frequency domain: **The Area Formula** (Bode/Horowitz)



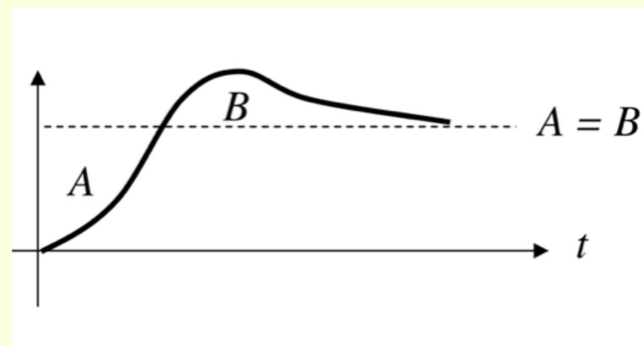
Isaac M. Horowitz (1920–2005)




$$\int_0^\infty log|S(j\omega)|d\omega = 0$$

(for L(s) with poles-zeros excess of 2 or more, and no open-loop poles in RHP)

Time domain: another "Area Formula"




$$\int_0^\infty e(t)dt = 0$$

(for L(s) having 2 or more integrators)

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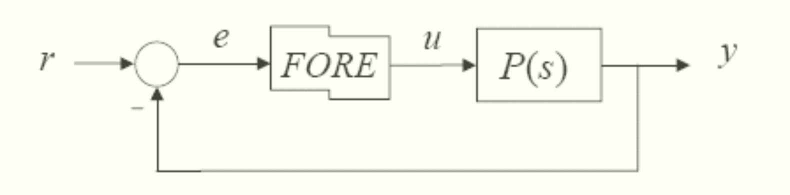
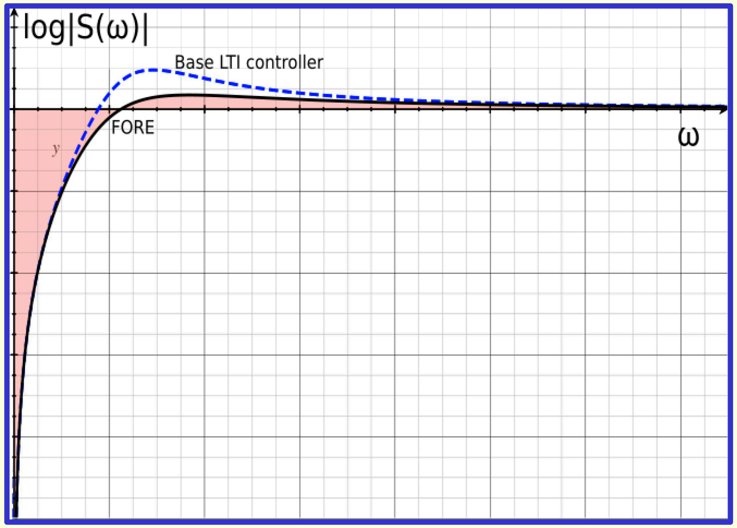
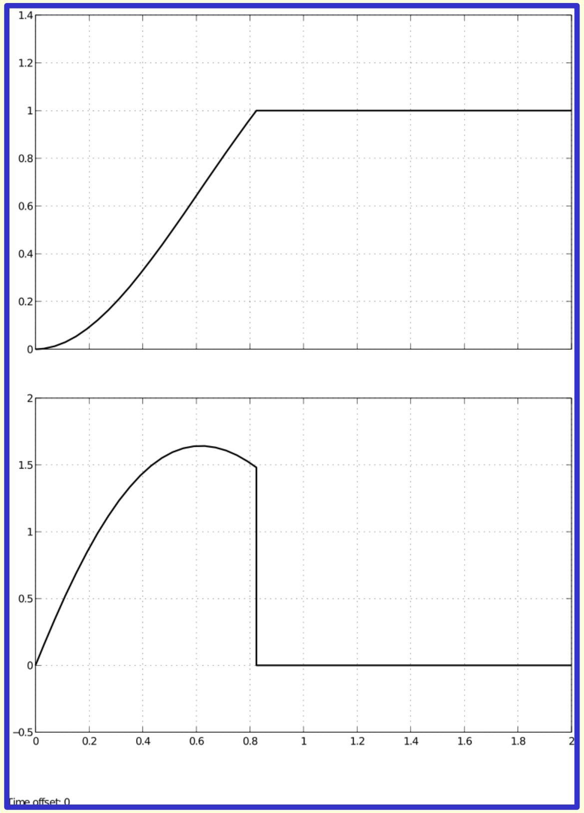
# Main motivation:
# Overcome Fundamental Limitations of LTI controllers



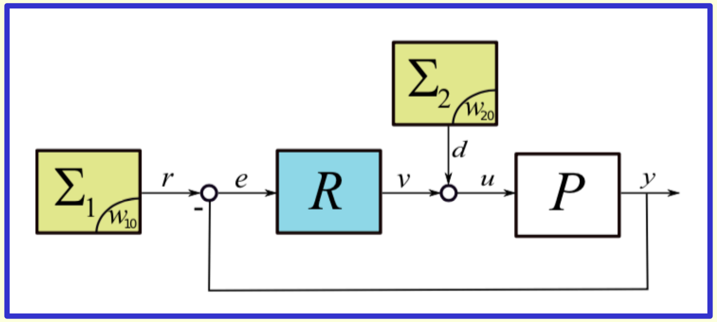

$$\int_0^\infty log|S(j\omega)|d\omega < 0 \ \text{(FORE)}$$

No overshoot (even for fast responses) !

$$\int_0^\infty log|S(j\omega)|d\omega \geq 0 \ \text{(any LTI controller)}$$

LTI controllers produce overshoot (bigger in faster responses) !

# A hybrid/impulsive control system




$$P: \begin{cases} \dot{\mathbf{x}}_p(t) = A_p\mathbf{x}_p(t) + B_r e(t) \\ y(t) = C_p\mathbf{x}_p(t) \end{cases}$$

$$R: \begin{cases} \dot{\mathbf{x}}_r(t) = A_r\mathbf{x}_r(t) + B_r e(t), & e(t) \neq 0 & \text{(Flow equation)} \\ \mathbf{x}_r(t^+) = A_\rho\mathbf{x}_r(t), & e(t) = 0 & \text{(Jump equation)} \\ v(t) = C_r\mathbf{x}(t) + D_r e(t) \end{cases}$$

- Base system (no resets) is LTI
- Zero-crossing resetting law: e(t) = 0 (other laws are possible as well)
- Reset events are (closed-loop**) state-dependent**
- Closed-loop system: **state-dependent** impulsive differential equation

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t), & \mathbf{x}(t) \notin \mathcal{M} & \text{(Flow equation)} \\ \mathbf{x}(t^+) = A_R\mathbf{x}(t), & \mathbf{x}(t) \in \mathcal{M} & \text{(Jump equation)} \end{cases}$$

- The reset set *M* defines reset instants,

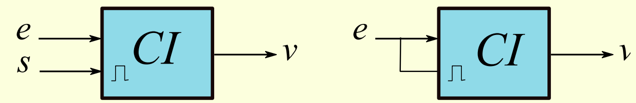
$$\mathcal{M} = \{\mathbf{x} \in \mathbb{R}^n : C\mathbf{x} = 0, (I - A_R)\mathbf{x} \neq 0\}$$

- And the after-reset set $M_R$

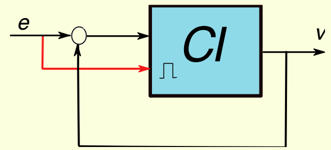
$$\mathcal{M}_R = A_R\mathcal{M}$$

# Algunos controladores reseteados

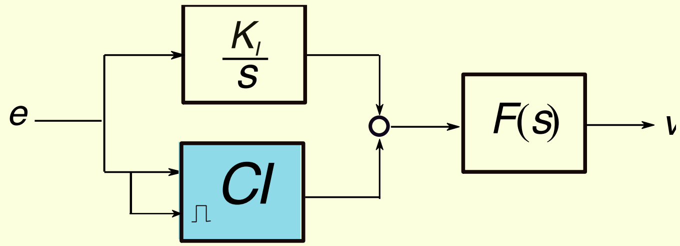
Clegg integrator (1958)




Horowitz FORE (1975)

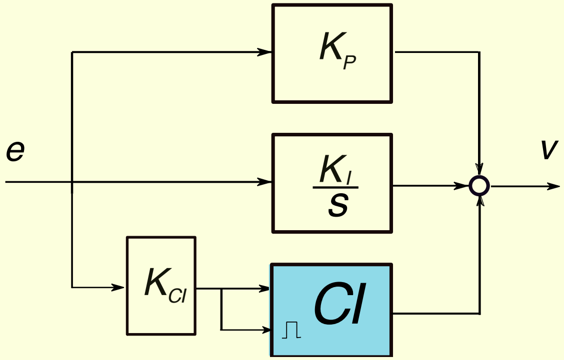



Horowitz reset controller (1974)




PI+CI (2007-2012)

Reset/Hold (2020)




Full reset/Partial reset

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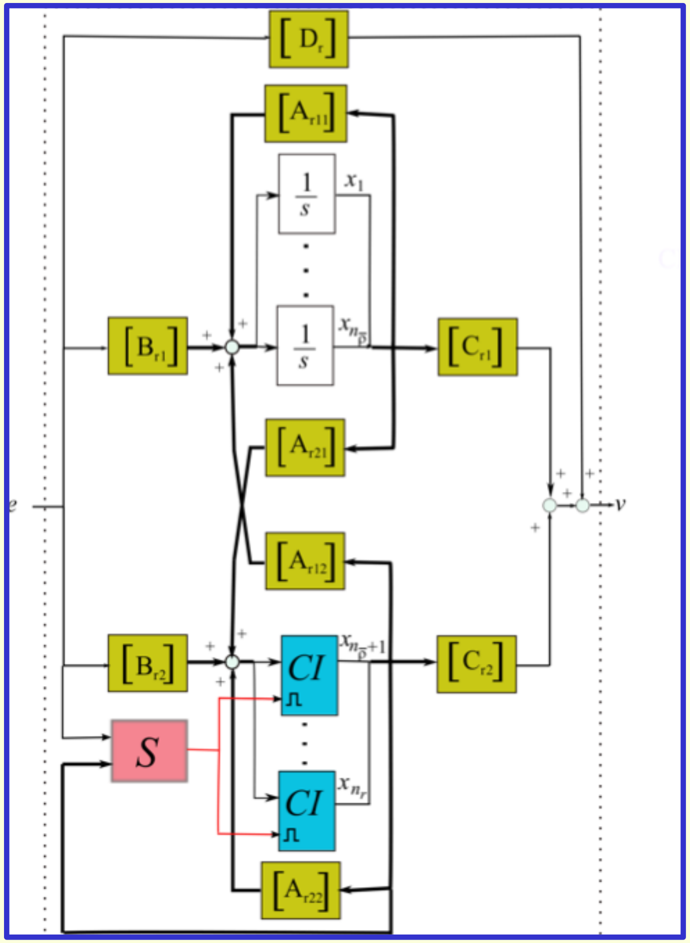
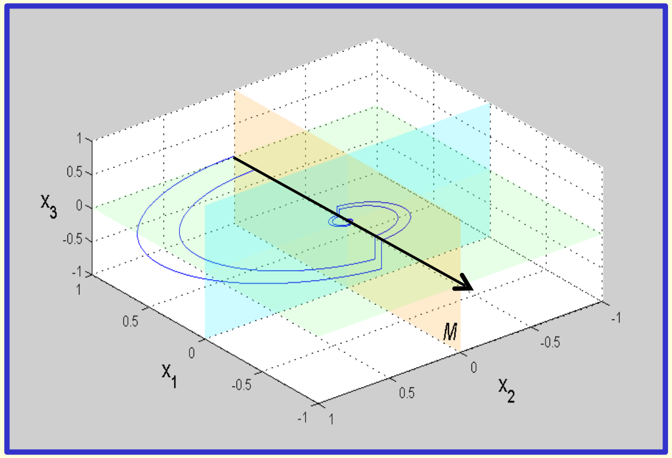
$$R: \begin{cases} \dot{\mathbf{x}}_r(t) = A_r\mathbf{x}_r(t) + B_r e(t), & e(t) \neq 0 \qquad \text{(Flow equation)} \\ \mathbf{x}_r(t^+) = A_\rho \mathbf{x}_r(t), & e(t) = 0 \qquad \text{(Jump equation)} \\ v(t) = C_r\mathbf{x}(t) + D_r e(t) & \end{cases}$$



# Stability (Lyapunov)

- A more general hybrid/impulsive dynamical system:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t), & C\mathbf{x}(t) \neq 0 \\ \mathbf{x}^+(t) = B\mathbf{x}(t), & C\mathbf{x}(t) = 0 \end{cases}$$




... is **stable** if for any ε > 0 there exist δ> 0 such as

$$\|\mathbf{x}_0\| \leq \delta \Rightarrow \|\mathbf{x}(t)\| \leq \epsilon, \text{ for any } t > 0$$

... is **attractive** if $\mathbf{x}(t) \to 0$ as $t \to \infty$

... is **asymptotically stable** if it is stable and attractive

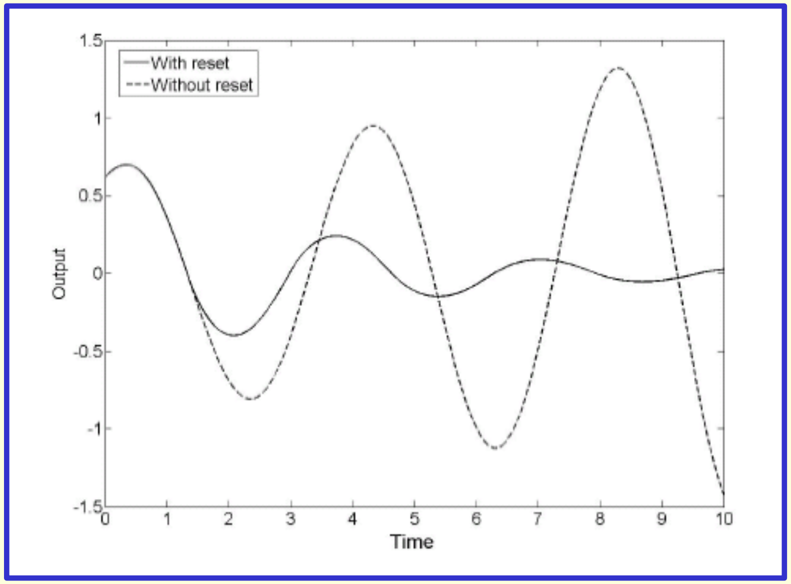
- The continuous base system (without reset actions, only flowing) is asymptotically stable if and only if ***A*** es Hurwitz-stable

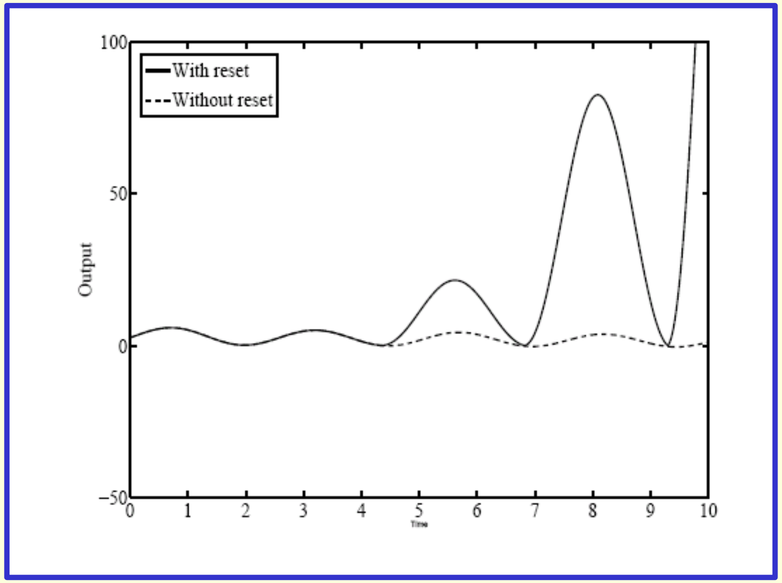
- The discrete system (without continuous dynamics, only jumping) is asymptotically stable if and only if ***B*** es Schur-stable

- **An open problem:** The hybrid/impulsive system, with parameters A, B, and C, is asymptotically stable if and only if ???

# Stability (Lyapunov)

The problem is **non-trivial**: reset can stabilize an unstable base system, … But also can destabilize a stable base system !




Some sufficient conditions for stability:

- independent on the reset instants: ***H***$_\beta$ **-condition (Bekker-Hollot-Chait'2000)**
- dependent on the reset instants**: "dwell-time"-based conditions**

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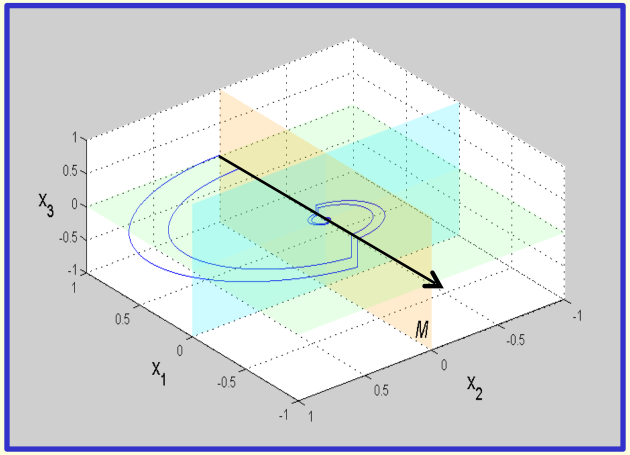
- For low order systems reset is periodic, but this is not true in general.
- In these simple cases, with constant reset interval $\Delta$, stability problem reduces to check if the matrix $A_R e^{A\Delta}$ is Schur-stable.




Example: Reset is periodic with period $\Delta = 3.16$, and in particular:

$$\mathbf{x}_1(k+1) = -0.30\mathbf{x}(k)$$

$$\lambda\{A_R e^{A\Delta}\} = \{-0.30, -0.73, 0\}$$

- If A is Hurwitz-stable then there always exists a **minimum** dwell-time such as the reset system is asymptotically stable
- if A is not Hurwitz-stable then the reset system is stable if the dwell-time is in some **interval**.

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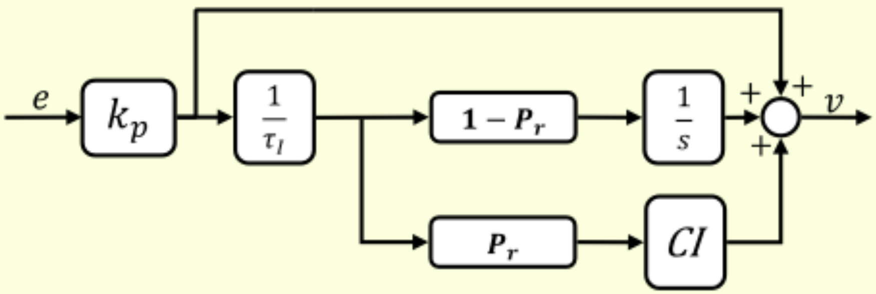
# 3. The PI+CI controller

(Baños and Vidal'2007-2012)




- A simple structure easily implementable, with few parameters
- Application target: process control
- Hopefully good transitory and steady state properties
- Also antiwindup behavior
- "Simple" tuning rules
  - Tune the base PI controller
  - Select the reset percentage to reduce overshoot
- A fast response with no excessive overshoot may be obtained, overcoming LTI compensation limitations.
- Very intuitive for manual tuning: reset appears as a single parameter $p_{\text{reset}}$
- CI: A "derivative" action without increasing cost of feedback

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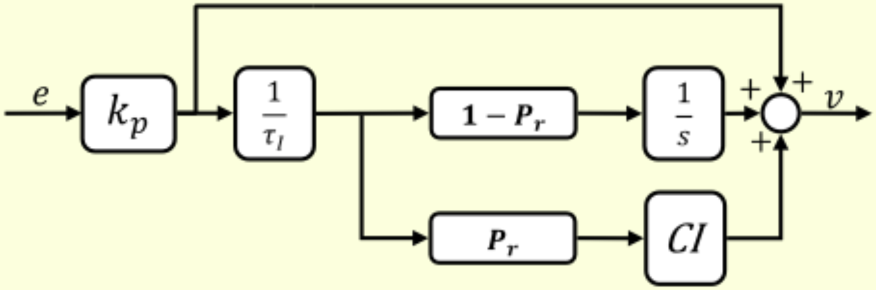
## First order system plus deadtime (FOPDT)

$$P(s) = \frac{k}{\tau s + 1} e^{-hs}$$

1. First, the base PI is tuned by using (for example) the IMC/SIMC rule:

$$k_p = \frac{\tau}{2kh}$$

$$\tau_I = 8h$$




2. Then, the parameter $p_r$ is tuned with ...:

- ... low values for "delay-dominant" systems
- ... middle-high values for "lag-dominant" systems
- ... $p_r = 1$ for integrating systems

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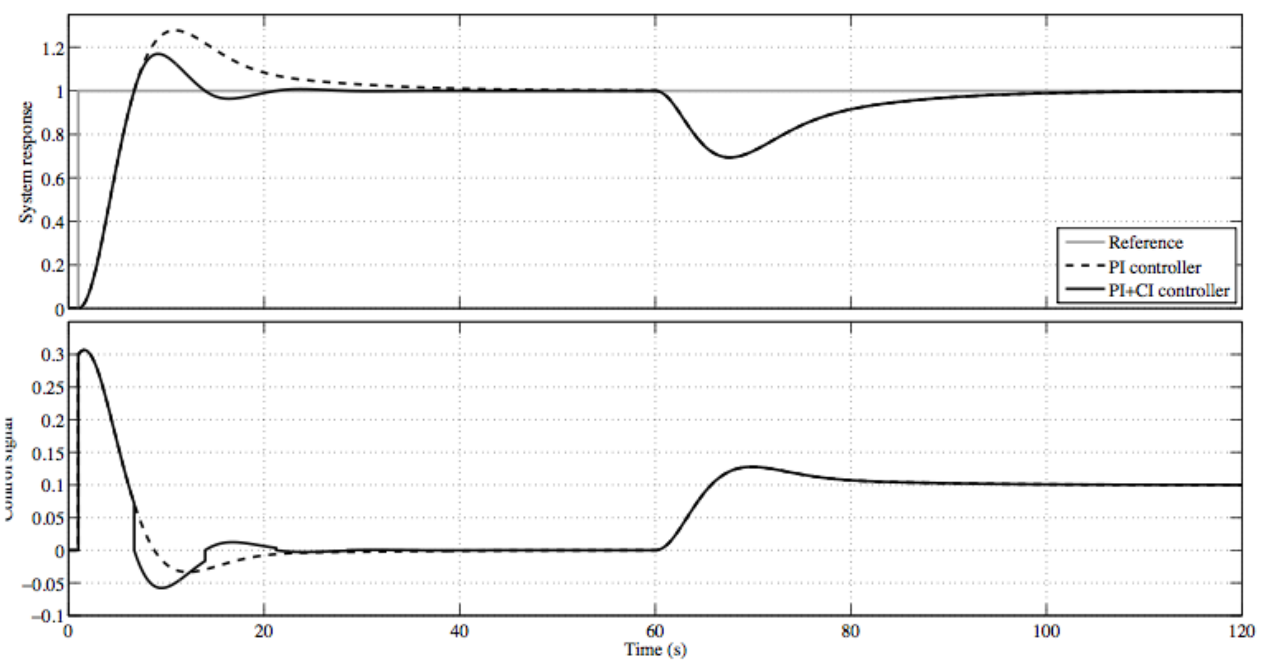
## Integrating plus deadtime systems (IPDT) – "integrating systems"

$$P(s) = \frac{1}{s} e^{-1.69s}$$

PI-base (SIMC): $k_p = 0.3, \tau_I = 13.5$

$p_r = 1$

| | Reference | | Disturbance | | Stability margins | |
|---|---|---|---|---|---|---|
| | IAE ($s$) | ITAE ($s^2$) | IAE ($s$) | ITAE ($s^2$) | $\varphi_m$ (°) | $A_m$ (dB) |
| PI | 6.43 | 62.80 | 4.48 | 328.3 | 48.6 | 24.28 |
| PI+CI | 4.17 | 20.44 | 4.48 | 328.3 | 48.4 | 23.9 |



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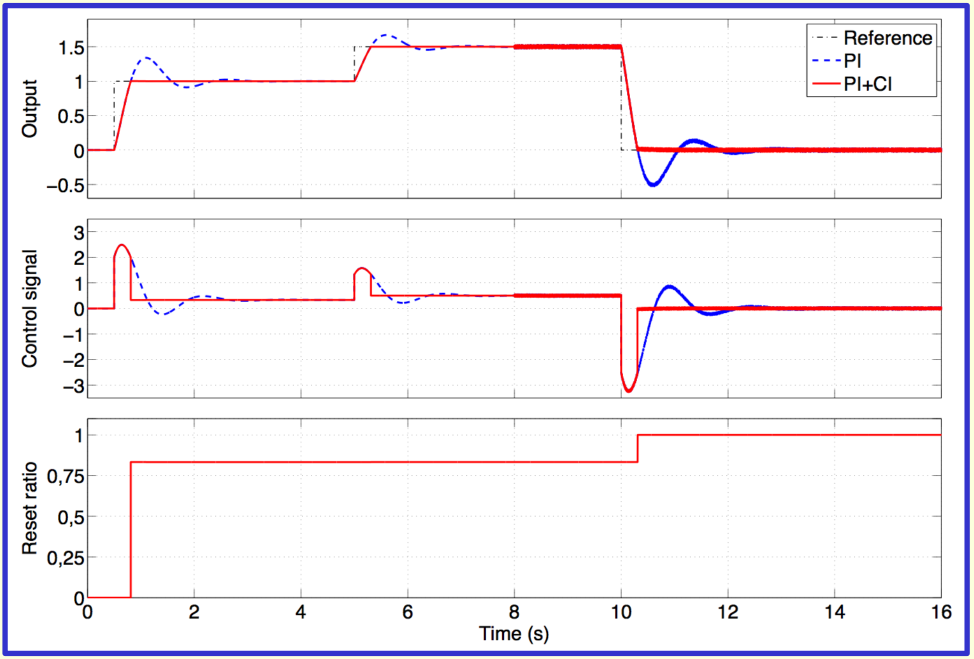
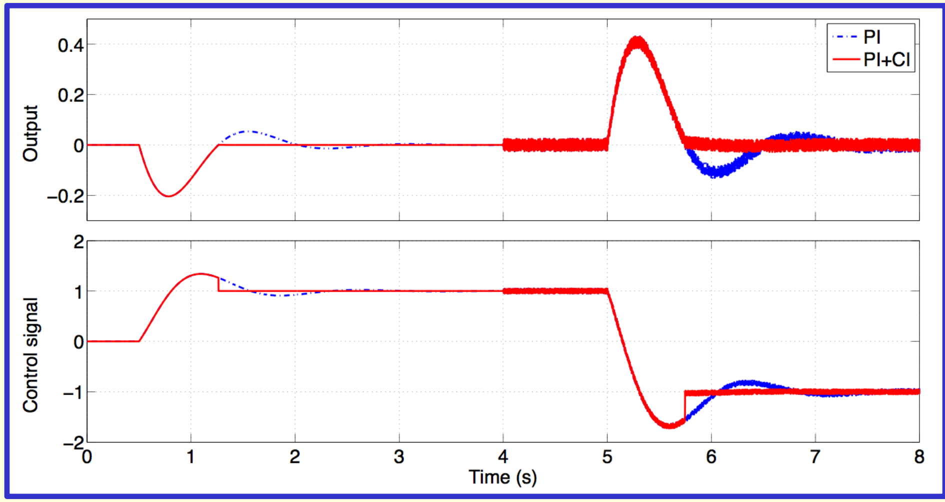
## Example: First order systems (reset band, time-varying reset rate)

$$P : \begin{cases} \dot{x}_p(t) = -a_0 x_p(t) + b_0 v(t) \\ y(t) = x_p(t) \end{cases}$$

**It is possible to obtain a finite settling-time !**



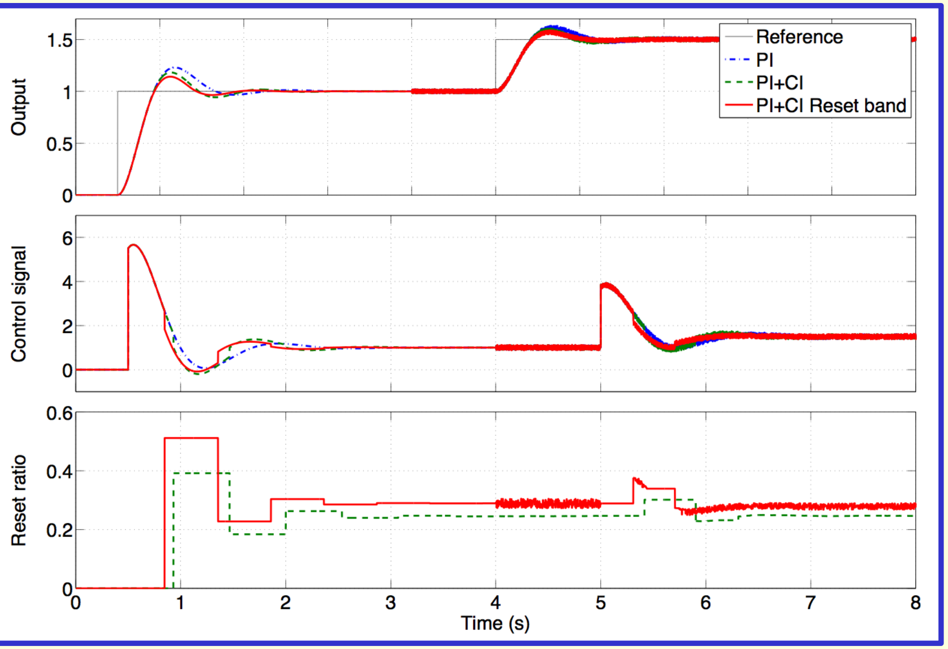




Step references tracking

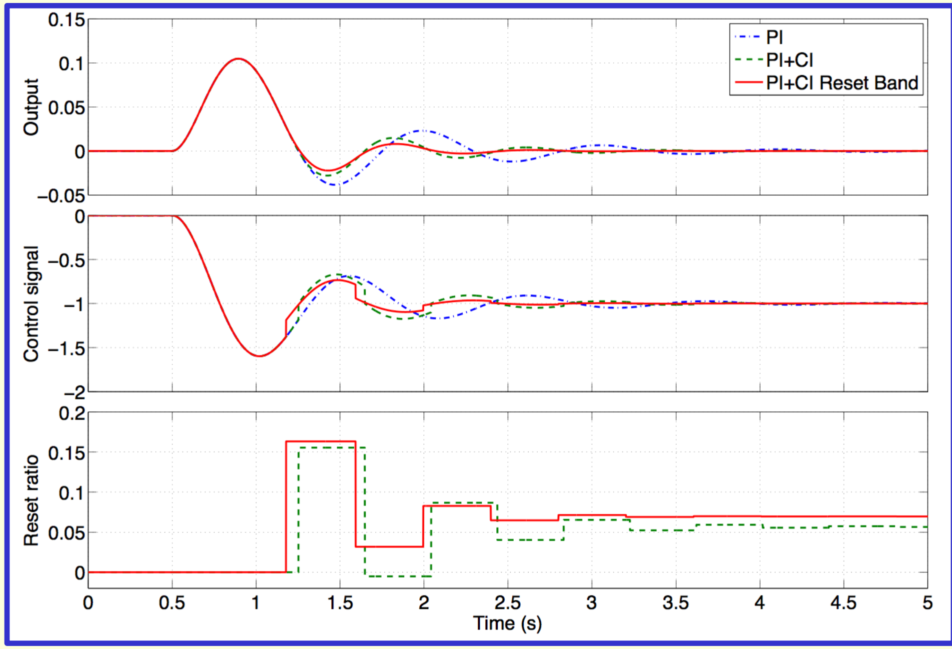
Step disturbances rejection

## Example: Second order systems (reset band, time-varying reset rate)

$$P(s) = \frac{b_0}{s^2 + a_1 s + a_0}$$




Step references tracking




Step disturbances rejection

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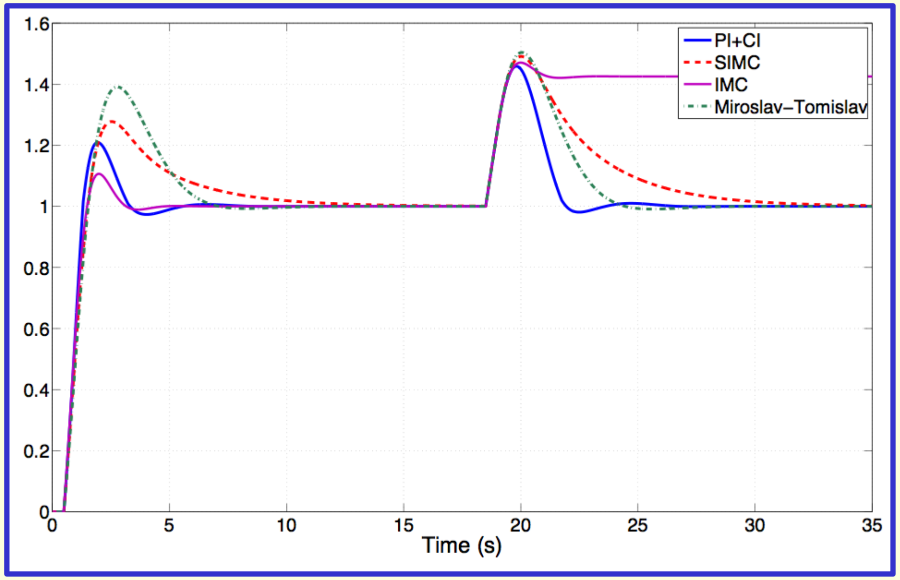
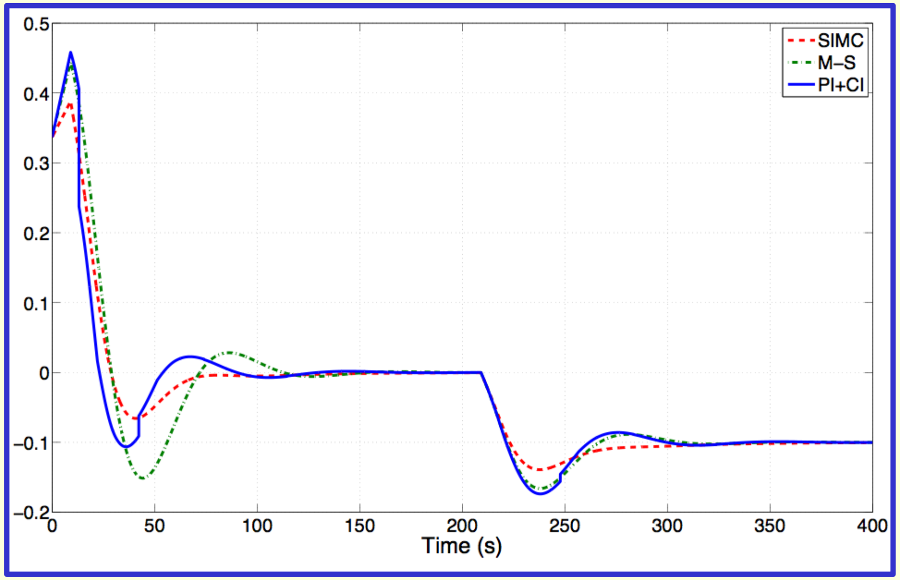
## Example: IPDT Systems (time-varying reset band, time-varying reset action)



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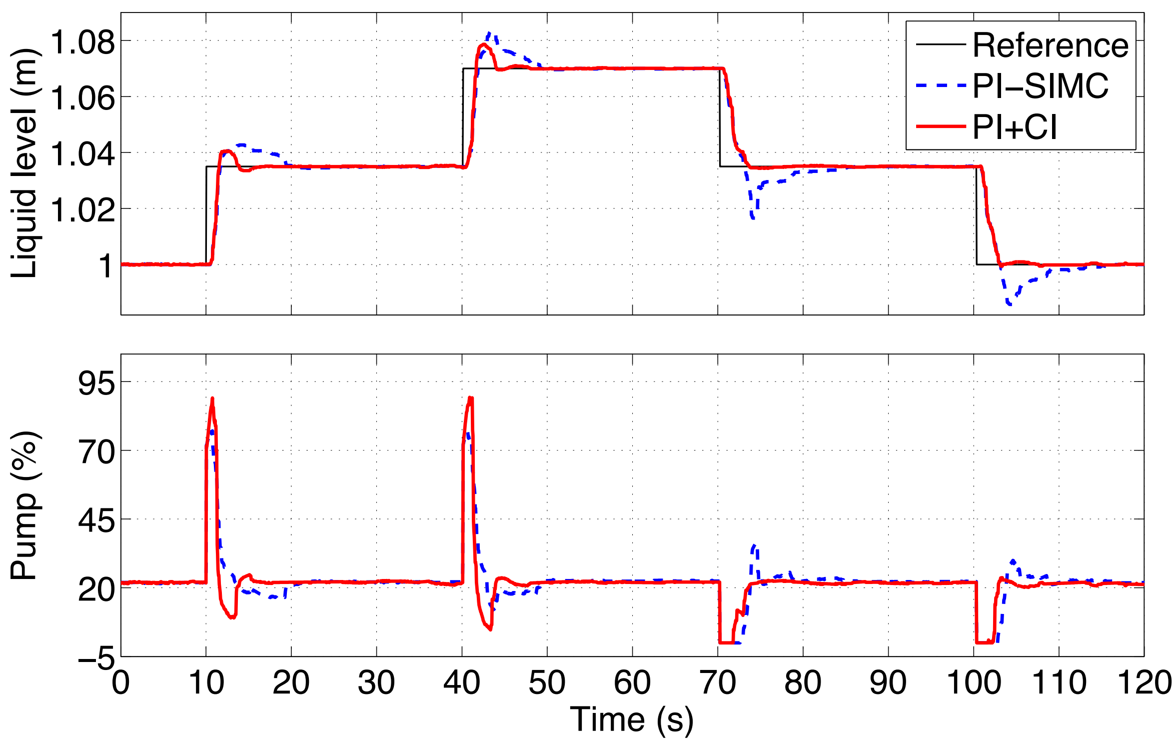
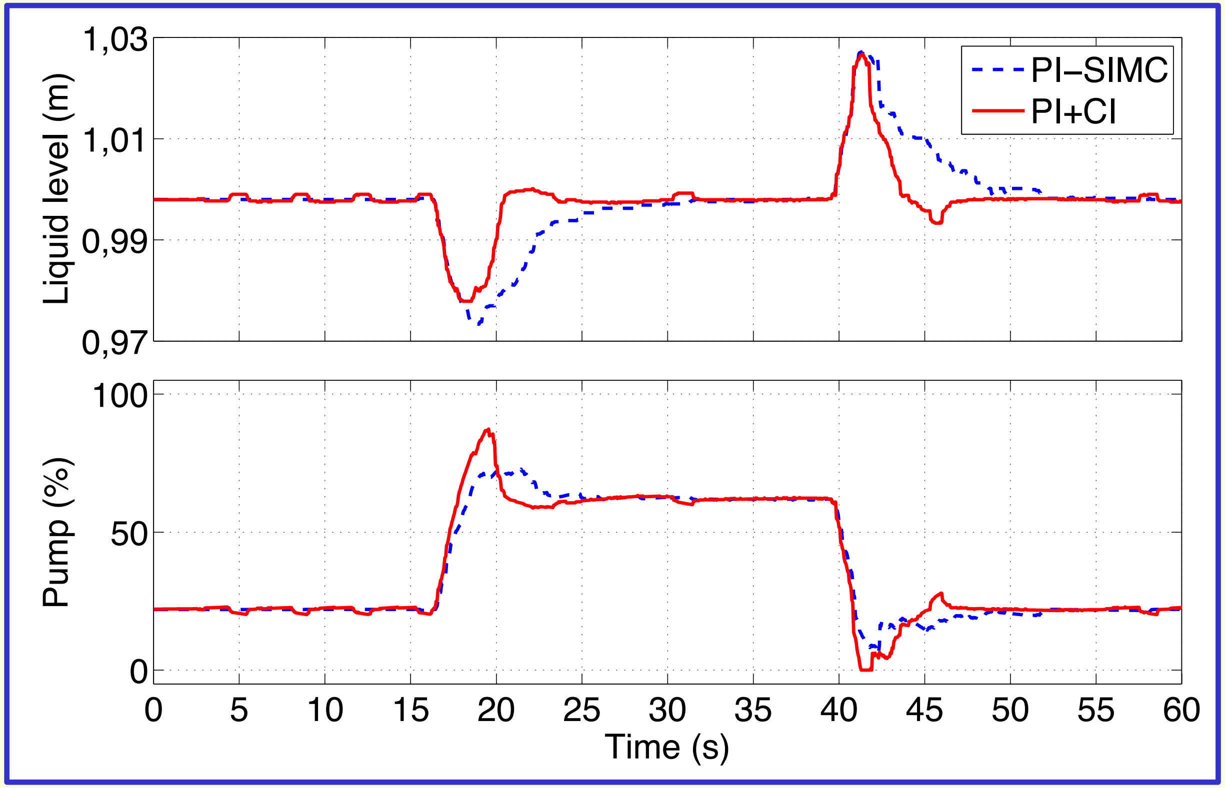
## Application: Level control system (variable reset band, variable reset action)




- IAE-Tracking error is reduced 40 % with respect to a well-tuned PI controller

## Application: Level control system (variable reset band, variable reset action)




- IAE-Disturbance rejection is improved a 50% with respect to a well-tuned PI

# A. Baños. 4. Ejemplos y aplicaciones

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| Aplicación | Laboratorio | Industria | Referencia |
|---|---|---|---|
| Supresión de vibraciones para estructuras flexibles | X | | (Bobrow *et al.*, 1995) |
| Control de velocidad de sistemas electromecánicos | X | | (Zheng *et al.*, 2000) |
| Control de discos duros | X | | (Wu et al., 2007)<br>(Guo *et al.*, 2011)<br>(Li *et al.*, 2011) |
| Propulsión marina | X | | (Bakkeheim et al., 2008) |
| Posicionamiento de actuadores piezoeléctricos | X | | (Zheng et al., 2007) |
| Teleoperación | X | | (Fernández et al., 2011)<br>(Falcón et. al., 2013) |
| Campos de colectores solares* | X<br>(Univ. Almería, Plataforma solar de Almería-CIEMAT) | | (Vidal et al., 2008) |
| Control de pH en línea* | X | | (Carrasco y Baños, 2012)<br>(Baños y Davó, 2014) |
| Control de temperatura en intercambiadores de calor* | X | | (Vidal y Baños, 2010)<br>(Moreno et al., 2013) |
| Control de nivel* | X | | (Davó y Baños, 2016) |
| Control de recirculación de gases de escape | X | | (Panni et al., 2014) |
| Cocinas de inducción** | X | X<br>(Universidad de Zaragoza, BSH Electrodomésticos) | (Paesa, 2011) |
| Control de grúas puente | X | | (Raimúndez et al. 2012) |
| Servomotores | X | | (HosseinNia et al., 2013) |
| Motores síncronos con excitación de imán permanente* | simulación<br>(Universidad de Vigo) | | (Delgado et al, 2014) |
| Control de *boost converters* * | X<br>(Universidad Politécnica de Cataluña) | | (Nair et al., 2018) |

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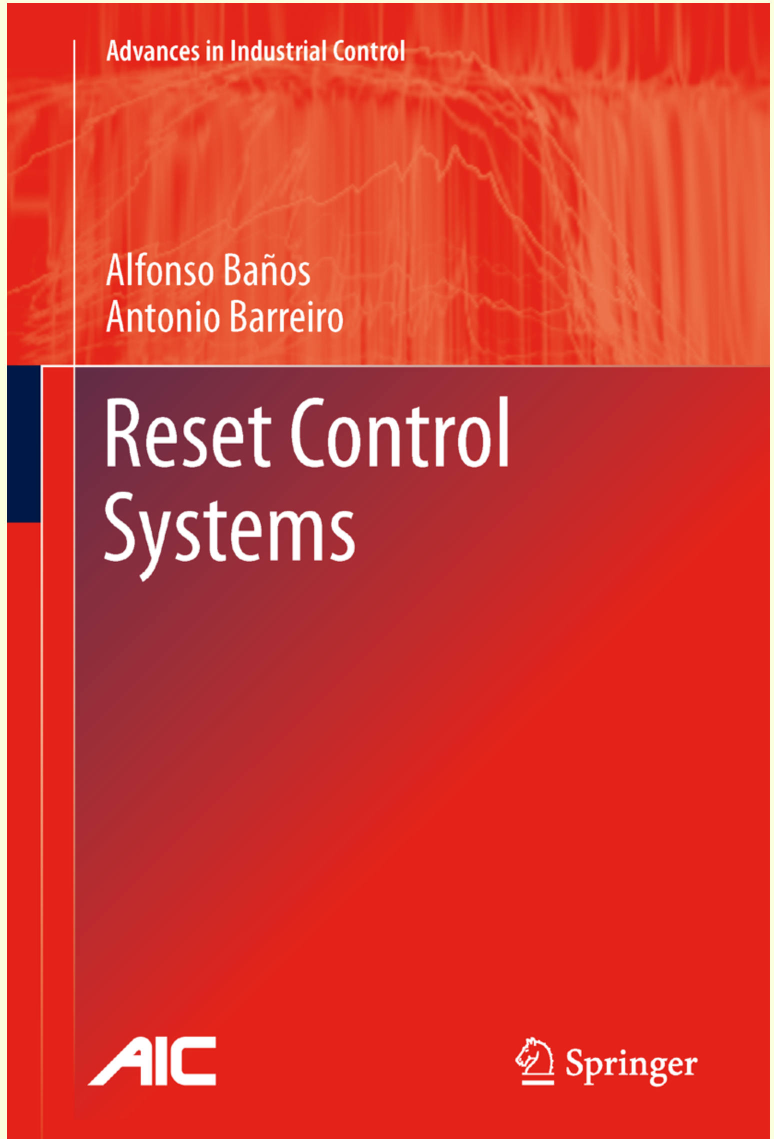
# CONCLUSIONS

- Reset is a simple way to improve control systems performance, overcoming LTI compensation fundamental limitations
- Stability is a main concern, since reset may have a destabilising effect
- It is not of general use, closed loop response must have overshoot for reset to be effective
- Reset control systems are hybrid systems, modelled as impulsive differential equations/hybrid dynamical systems/hybrid automata
- Reset may be modelled as state-dependent event
- PI+CI has been found very effective for lag dominant and integrating systems
- Improvements may be achieved by a variable reset band and a variable reset ratio. Also with reset and hold strategies for delay-dominant systems
- **Many open directions for research, both in theory and practice**


Advances in Industrial Control
Alfonso Baños
Antonio Barreiro
Reset Control Systems
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