Distributed dynamic consensus in multi-robot systems

Rosario ARAGUES

raragues@unizar.es

Universidad de Zaragoza

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Universidad Zaragoza 1542

Departamento de Informática e Ingeniería de Sistemas Universidad Zaragoza 1542

ROBOTICS UNIVERSIDAD DE ZARAGOZA

Instituto Universitario de Investigación en Ingeniería de Aragón Universidad Zaragoza

Organización

- Introduction
- The consensus problem
- Dynamic consensus for map merging
- Dynamic consensus for multi-leader formation control
- Consensus for intermittent connectivity
- Conclusions

Organización

- ❑ **Introduction**
- ❑ The consensus problem
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- ❑ Conclusions

Distributed dynamic consensus in MRS - Rosario ARAGUES

Introduction

An example

With 10 robots, with 100 robots, with 1,000 robots?

Kaveh Fathian, Sleiman Safaoui, Tyler Summers, Nicholas Gans
University of Texas at Dallas

https://youtu.be/AxT-fFcGQoA

K. Fathian, S. Safaoui, T. H. Summers and N. R. Gans, "Robust Distributed Planar Formation Control for Higher Order Holonomic and Nonholonomic Agents," in IEEE Transactions on Robotics, doi: 10.1109/TRO.2020.3014022.

Introduction

Examples of collective motions nature?

Flock of Starlings (National Geographic)

https://www.youtube.com/watch?v=V4f_1_r80RY&t=10s

School of fish (Wikipedia)

BBC Earth https://youtu.be/15B8qN9dre4?t=48

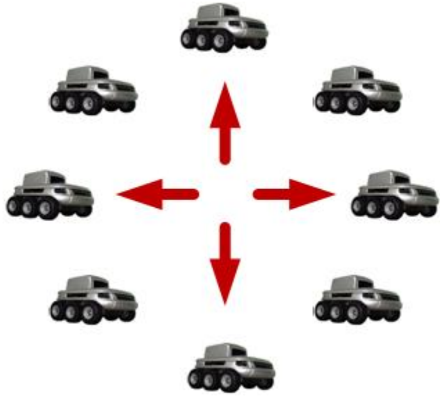
Herd of sheep

https://www.youtube.com/watch?v=tDQw21ntR64

Introduction

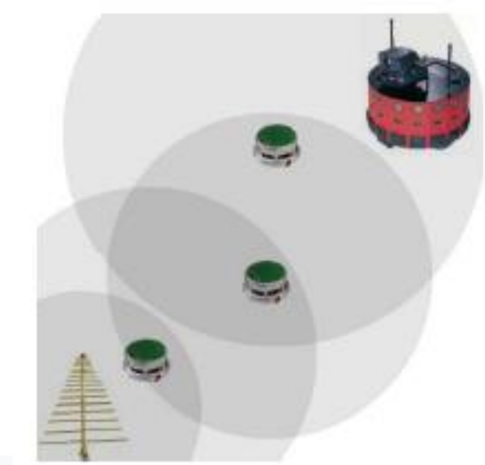
Properties of collective motions

- Collective motion in nature:
  - Local interaction rules
  - No collisions. Reactivity to obstacles
  - No apparent leader. No central point of failure (increased robustness)
  - Coalescing and splitting
  - Different species have different flocking characteristics
  - Benefits: energy saving (e.g., geese extend flight range by 70%); signs of better navigation accuracy

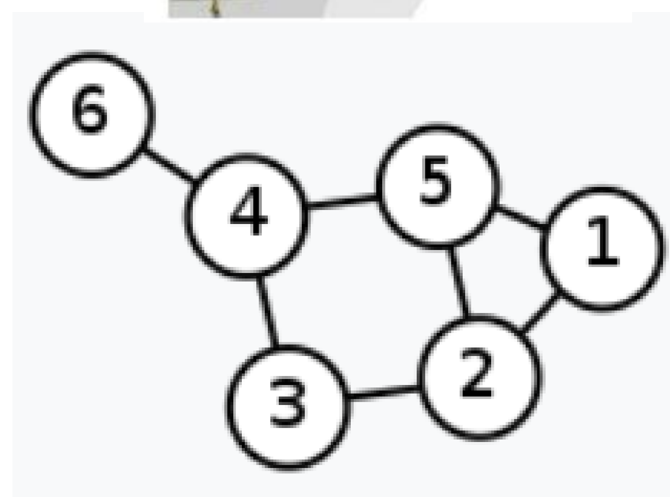


Framework multi-robot systems (MRS)

Control, perception, decision making, navigation, coordination



- Terms used: robot swarms / robot teams / robot networks
- Distributed nature of many problems and applications
- Increased overall performance: extends the capabilities of what can be done with a single robot
- Redundancy and increased robustness
- Graphs

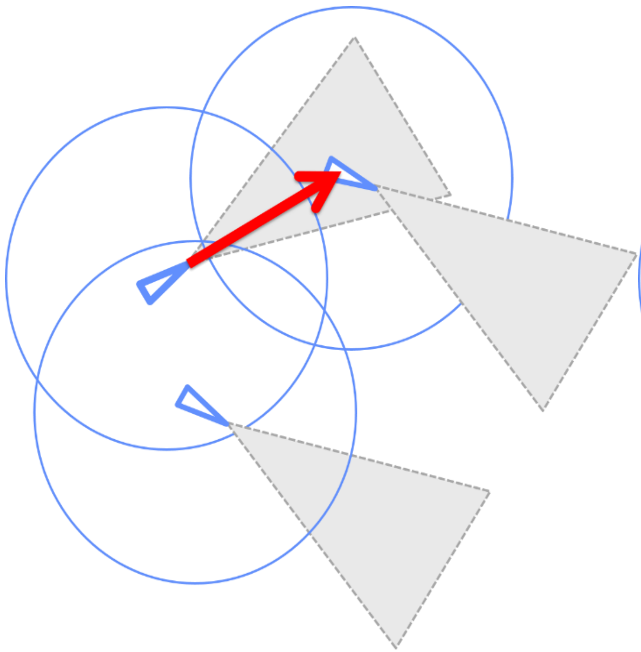


- Challenges: coordinating the team, make decisions on partial and different data, communication..

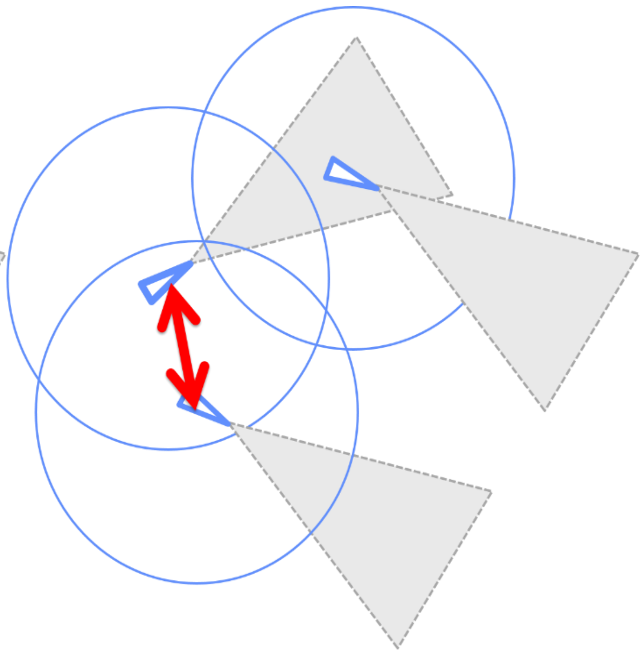
Introduction

Explicit / implicit communication (Sensing vs. Sending data)

Examples of sensing (limited field of view, gray areas) and comm. (blue circular regions)



Sensing

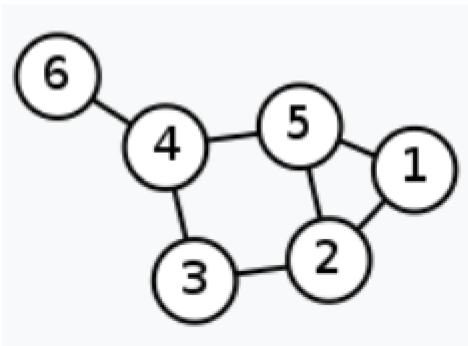


Communication

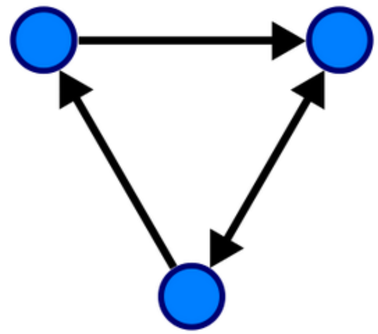
- **Sensing graphs**: for each sensors, encode what robots can be locally sensed
- **Communication graphs**: for each communication medium, encode with which robots a comm. link can be established (uni- or bi-directional)

Introduction

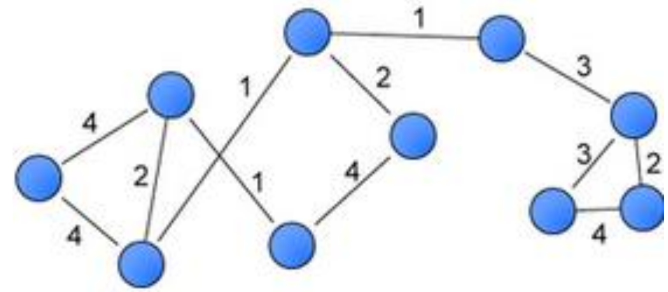
Graphs



Undirected graph



Directed graph



Weighted graph

- Fixed vs. time varying
- Synchronous, asynchronous, event-triggered, gossip (randomized)

Mesbahi, Mehran, and Magnus Egerstedt. **Graph Theoretic Methods in Multiagent Networks**. PRINCETON; OXFORD: Princeton University Press, 2010. www.jstor.org/stable/j.ctt1287k9b Accessed July 10, 2020. doi:10.2307/j.ctt1287k9b.

Introduction

Undirected Graphs

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ Graph

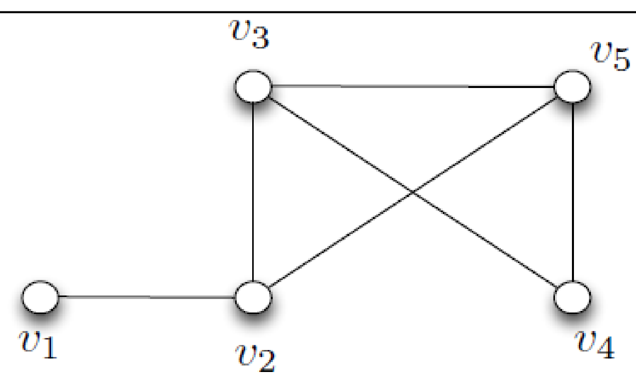
Nodes, vertex set (e.g, robots) $\mathcal{V} = \{v_1, \ldots, v_N\}$

Edges (e.g. comm. / sensing between robots)

$$\mathcal{E} \subseteq \{(v_i, v_j)\}, i = 1 \ldots N, j = 1 \ldots N, i \neq j$$

Undirected:

$$(v_i, v_j) \in \mathcal{E} \Rightarrow (v_j, v_i) \in \mathcal{E}$$



$$\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$$

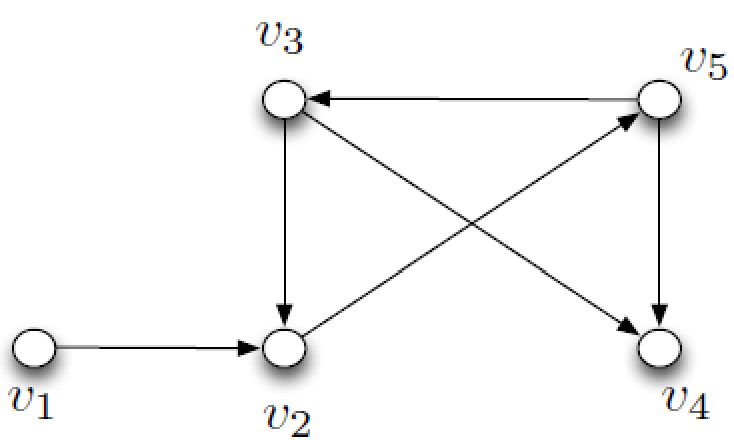
$$\mathcal{E} = \{(v_1, v_2), (v_2, v_3), (v_2, v_5), (v_3, v_5), (v_3, v_4), (v_4, v_5)\}$$

Directed Graphs

- Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- Nodes, vertex set (e.g, robots) $\mathcal{V} = \{v_1, \ldots, v_N\}$
- Edges (e.g. comm. / sensing between robots)

$$\mathcal{E} \subseteq \{(v_i, v_j)\}, i = 1 \ldots N, j = 1 \ldots N, i \neq j$$

- **Directed**: $(v_i, v_j) \in \mathcal{E} \nRightarrow (v_j, v_i) \in \mathcal{E}$



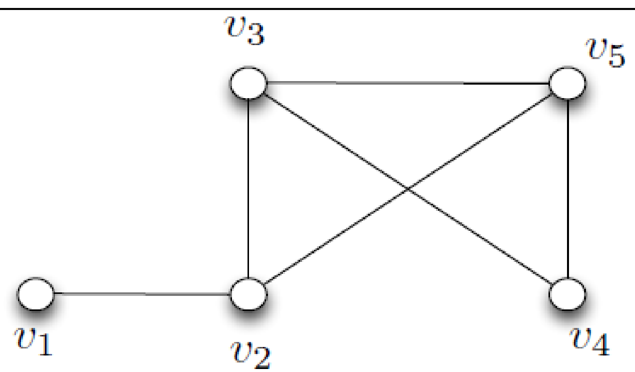
$$\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$$

$$\mathcal{E} = \{(v_1, v_2), (v_3, v_2), (v_2, v_5), (v_3, v_4), (v_5, v_3), (v_5, v_4)\}$$

Definitions

- **Neighbors** (set of neighbors) $\mathcal{N}_i = \{v_j \in \mathcal{V} | \, (v_j, v_i) \in \mathcal{E}\}$
- **Degree** of a node (undirected graphs) $d_i = |\mathcal{N}_i|$
- **In-degree, out-degree** of a node (directed graphs)
- **Path**: sequence of distinct vertexes such that the vertexes and are adjacent (neighbors)

$v_{i_0} v_{i_1} \dots v_{i_m}$ $\quad \forall k = 0, \dots, m-1 \quad$ v_{i_k} and $v_{i_{k+1}}$ are neighbors



$\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$

$\mathcal{E} = \{(v_1, v_2), (v_2, v_3), (v_2, v_5), (v_3, v_5), (v_3, v_4), (v_4, v_5)\}$

The neighbors of v3 are {v2, v4, v5}.
The degree of v3 is d3=3. The degree of v4 is d4=2.
There is (at least) a path between v1 and v5. E.g.: v1, v2, v3, v5.

Definitions

An **undirected** graph is said **connected** if there exists a path joining any two nodes

A **directed** graph is said **strongly connected** if there exists a (directed) path joining any two nodes

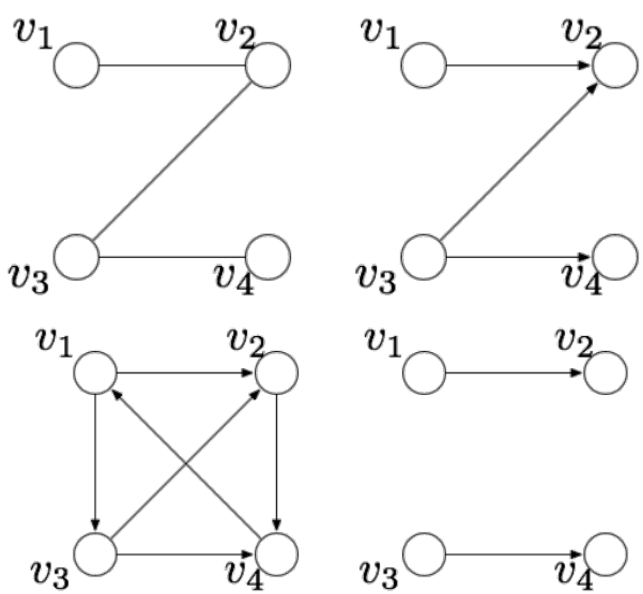
A directed graph is said **weakly connected** if there exists an undirected path joining any two nodes

connected (undirected)

weakly connected, but not strongly (directed)

strongly connected (directed)

Disconnected (directed)



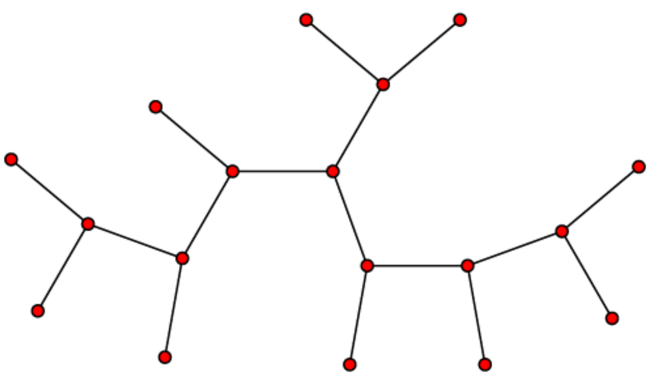
Special graphs / subgraphs

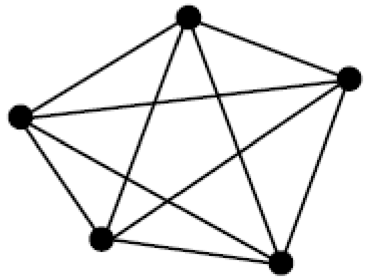
Trees (and spanning Trees): N nodes, N-1 edges, connected

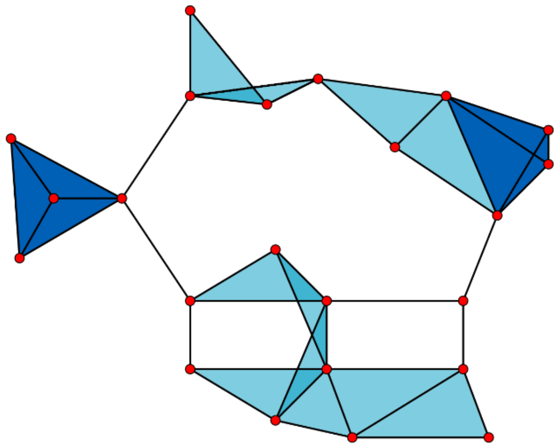
Complete graphs (all–to–all, fully connected), cliques

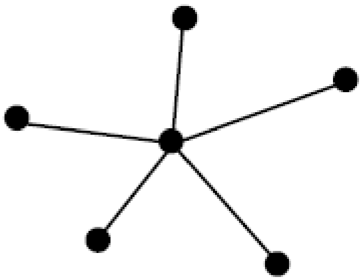
Star topology

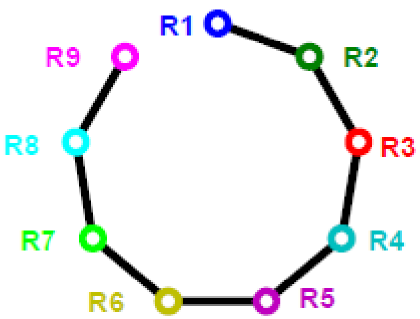
Line graph (path)











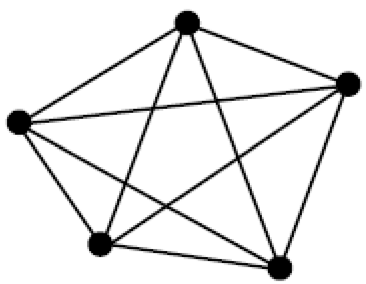
More on graphs, connectivity…

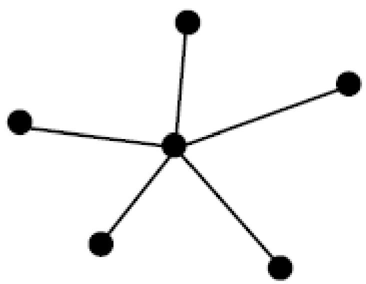
- Switching (time-varying)
- Random
- Jointly connected
- Interval of joint connectivity
- Synchronous / asynchronous, gossip
- Structure:
  - Globally reachable node
  - Rooted spanning trees
  - Regular graphs (all nodes with the same number of neighbors)
  - Lattice graph (mesh graph, or grid graph): regular tiling
- Weighted graphs
- Minimum-distance spanning trees (MST)
- (…)

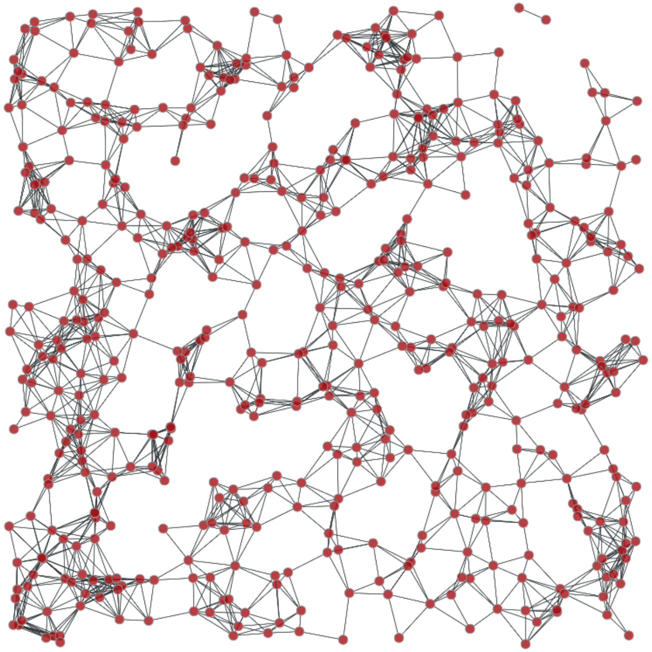
Centralized vs Distributed

- Multi-robot systems: every unit (robot) has:
  - limited sensing/communication (information **gathering**)
  - limited computing power (information **processing**)
  - limited available memory (information **storage**)
- **Centralized**: one unit communicates with all robots to issue commands
  - Single-point failure
  - Robots usually need the gathered information to run its local controller.
  - If the whole state of all the robots is needed: increases with the number of robots
  - It may become unfeasible!

What is more appropriate here?







Introduction

Thus.. centralized or distributed?

automated warehouses

automated mobility-on-demand

search & rescue / surveillance

connected autonomous vehicles

Amanda Prorok (Univ. Cambridge). Summer School on Multi-Robot Systems 2019. ”Control and coordination”. Slides: http://mrs.felk.cvut.cz/summer-school-2019/assets/pdf/amanda.pdf

Algebraic graph theory (Graphs & Matrices)

Several matrixes can be associated to graphs and….

….several graph properties deduced from the associated matrices

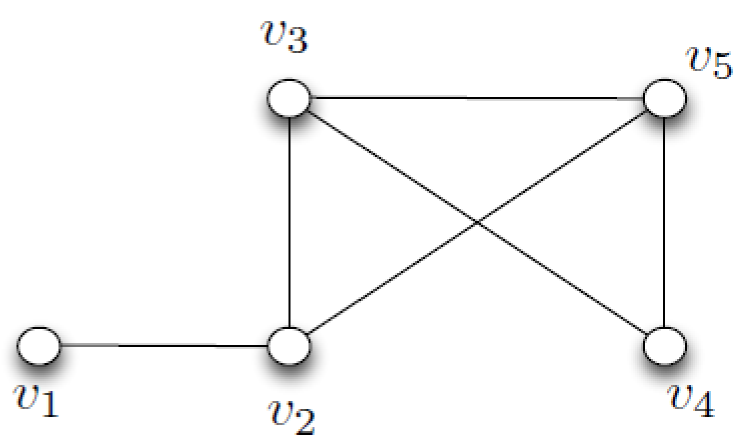
Algebraic tools **fundamental** for linking Graph Theory to the study of multi-robot systems

Adjacency Matrix

Degree Matrix

Incidence Matrix

Laplacian Matrix



Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

Introduction

Algebraic graph theory (Graphs & Matrices)

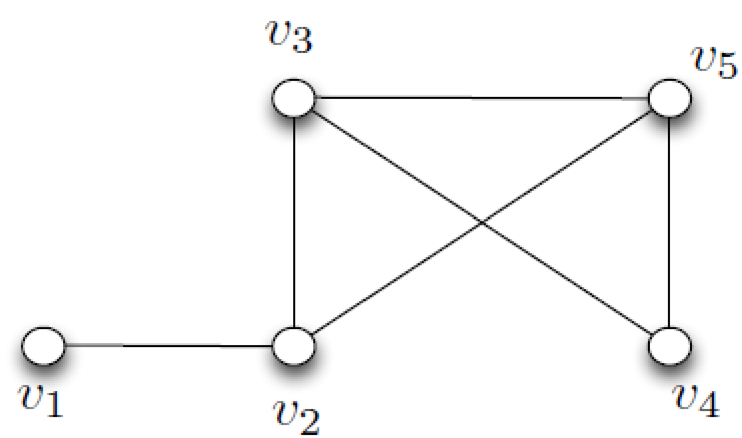
Several matrixes can be associated to graphs and….

….several graph properties deduced from the associated matrices

Algebraic tools **fundamental** for linking Graph Theory to the study of multi-robot systems

Adjacency Matrix $A \in \mathbb{R}^{N \times N}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$



Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

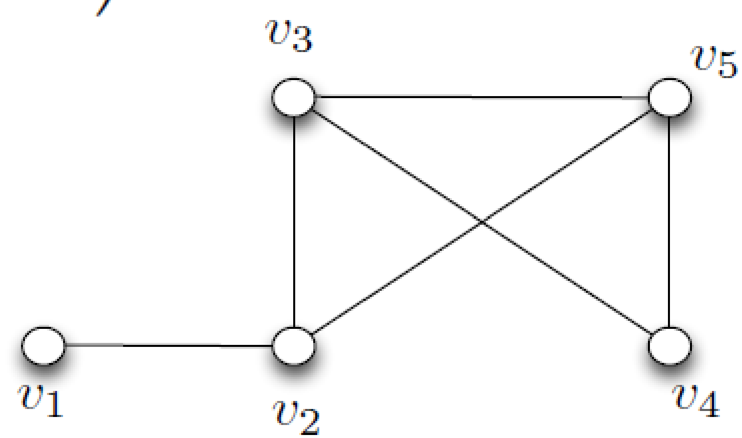
Introduction

Algebraic graph theory (Graphs & Matrices)

- Degree matrix $\Delta \in \mathbb{R}^{N \times N}$
- Degree (number of neighbors) of every node (robot):

$$\Delta = diag(d_i) \qquad \Delta = diag\left(\sum_{j=1}^{N} A_{ij}\right)$$

$$\Delta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

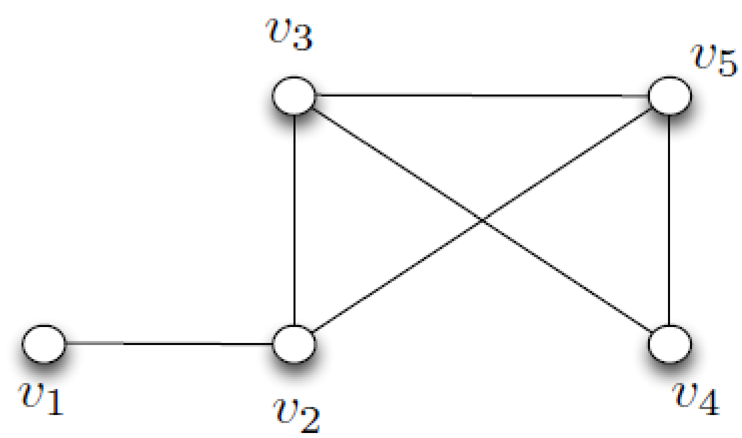


Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

Introduction

Laplacian matrix

Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



- **Laplacian** matrix $L \in \mathbb{R}^{N \times N}$

$$L = \Delta - A$$

Degree and Adjacency matrices

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{bmatrix}$$

Properties:

$$L\mathbf{1} = 0$$

$$0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$

$\mathcal{G}$ **connected** if and only if $\lambda_2 > 0$

Organización

- ❑ Introduction
- ❑ **The consensus problem**
- ❑ Dynamic consensus for map merging
- ❑ Dynamic consensus for multi-leader formation control
- ❑ Consensus for intermittent connectivity
- ❑ Conclusions

The consensus problem

- One of the most fundamental problem in multi-robots (and multi-agents) literature

- ***The consensus problem*: the goal and the rules**
  - Consider N robots with internal **state** $x_i \in \mathbb{R}$
  - Consider an internal **dynamics** for the state evolution. Here, single integrator:

    $$\dot{x}_i = u_i$$

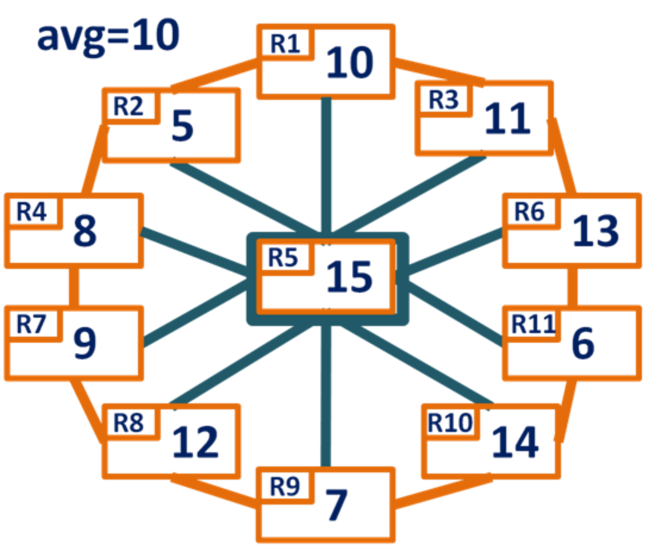
  - Consider an interaction **graph** between robots $\mathcal{G}$
  - Problem: design the control inputs u_i
  - so that all the sates **agree** on the same common value (unspecified, unknown, often the **average of xi(0)**)

    $$\lim_{t \to \infty} x_i(t) = \bar{x}, \ \forall i$$

  - by making use of only information from **neighbors** (decentralized)

The consensus problem. Any ideas?

- Several possibilities, some of them very intuitive



- Computational / storage / communication costs? (per iteration)
- Time until a robot gets the **average** value?
- What if the graph changes along time?
- Key idea of the consensus protocol (next): **distributed**, **scalable**

The consensus problem

Several solutions. The most popular ones:

- **The consensus protocol in discrete time**: Iteratively, each robot:
- (based on the robot degree:) $$x_i[t+1] = \frac{1}{|\mathcal{N}_i|+1}(x_i[t] + \sum_{j \in \mathcal{N}_i} x_j[t])$$
- (Metropolis weights:) $$x_i(t+1) = W_{ii}(t)x_i(t) + \sum_{j \in \mathcal{N}_i(t)} W_{ij}(t)x_j(t),$$

$$W_{ij}(t) = \begin{cases} \frac{1}{1+\max\{d_i(t), d_j(t)\}} & \text{if } \{i,j\} \in \mathcal{E}(t), \\ 1 - \sum_{\{i,k\} \in \mathcal{E}(t)} W_{ik}(t) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

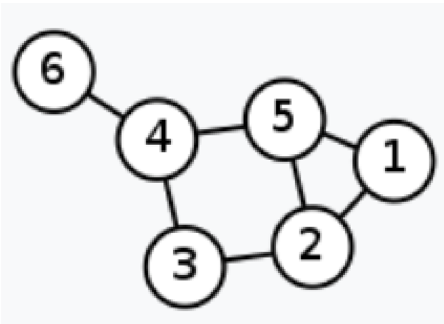
- (Laplacian -> Perron matrix:) $$x_i(k+1) = x_i(k) + u_i(k), \text{ for } k \geq 0$$

$$u_i = \alpha \sum_{j \in \mathcal{N}_i} (x_j - x_i)$$ with α positive 0 < α < 1/(2N)

- Results for **undirected** graphs: Asymptotic convergence to the **average** of the initial robot states if the graph is **connected**

The consensus problem

An example: Metropolis weights



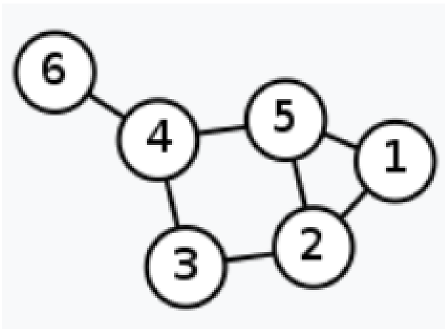
$$x_i(t+1) = W_{ii}(t)x_i(t) + \sum_{j \in \mathcal{N}_i(t)} W_{ij}(t)x_j(t), \quad i = 1, \ldots, n.$$

$$W_{ij}(t) = \begin{cases} \frac{1}{1+\max\{d_i(t), d_j(t)\}} & \text{if } \{i,j\} \in \mathcal{E}(t), \\ 1 - \sum_{\{i,k\} \in \mathcal{E}(t)} W_{ik}(t) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Robot	Initial state	Neighbors	Degre e	Weights (Metropolis)
i=1	x1(0)=5	N1={2,5}	d1=2	W12=1/4, W15=1/4, W11=0.5
i=2	x2(0)=20	N2={1,3,5}	d2=3	W21=1/4, W23=1/4, W25=1/4, W22=0.25
i=3	x3(0)=12	N3={2,4}	d3=2	W32=1/4, W34=1/4, W33= 0.5
i=4	x4(0)=2	N4={3,5,6}	d4=3	W43=1/4, W45=1/4, W46=1/4, W44=0.25
i=5	x5(0)=3	N5={1,2,4}	d5=3	W51=1/4, W52=1/4, W54=1/4, W55=0.25
i=6	X6(0)=22	N6={4}	d6=1	W64=1/4, W66=0.75
n=6	avg(0)=10.7			(the remaining weights equal 0)

The consensus problem

An example: Metropolis weights



$$x_i(t+1) = W_{ii}(t)x_i(t) + \sum_{j \in \mathcal{N}_i(t)} W_{ij}(t)x_j(t), \quad i = 1, \ldots, n.$$

$$W_{ij}(t) = \begin{cases} \frac{1}{1+\max\{d_i(t), d_j(t)\}} & \text{if } \{i,j\} \in \mathcal{E}(t), \\ 1 - \sum_{\{i,k\} \in \mathcal{E}(t)} W_{ik}(t) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

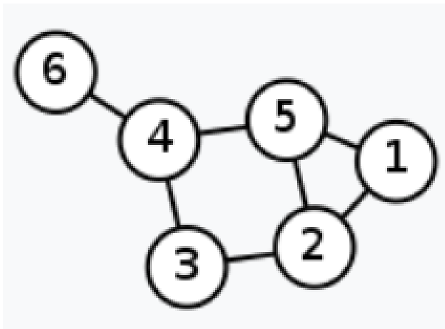
Robot i=1 (step t)
Send x1(t) to neighbors N1={2,5}
Receive x2(t) and x5(t) from neighbors
Update
x1(t+1)=0.5 * x1(t) + 0.25*x2(t) + 0.25*x5(t)

Robot i=6 (step t)
Send x6(t) to neighbor N6={4}
Receive x4(t) from neighbor
Update
x6(t+1)=0.75 * x6(t) + 0.25*x4(t)

Robot	Initial state	Neighbors	Degree	Weights (Metropolis)
i=1	x1(0)=5	N1={2,5}	d1=2	W12=1/4, W15=1/4, W11=0.5
i=6	X6(0)=22	N6={4}	d6=1	W64=1/4, W66=0.75

The consensus problem

An example: Metropolis weights

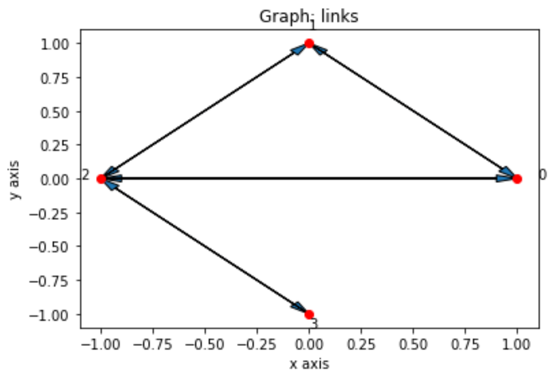


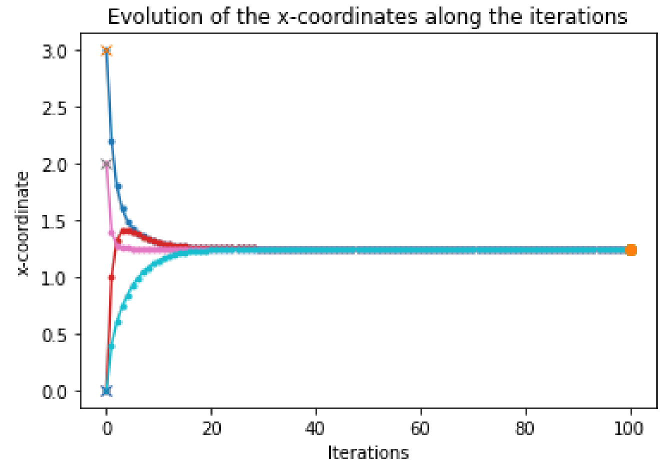
$$x_i(t+1) = W_{ii}(t)x_i(t) + \sum_{j \in \mathcal{N}_i(t)} W_{ij}(t)x_j(t), \quad i = 1, \dots, n.$$

$$W_{ij}(t) = \begin{cases} \frac{1}{1+\max\{d_i(t), d_j(t)\}} & \text{if } \{i,j\} \in \mathcal{E}(t), \\ 1 - \sum_{\{i,k\} \in \mathcal{E}(t)} W_{ik}(t) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Robot	State (t=0)	t=1	t=2	t=3	t=4	t=5	t=6	…	t=10
i=1	x1(0)=5	8.25	8.5	8.8	9.1	9.4	9.6	…	10.2
i=2	x2(0)=20	10	9.3	9.3	9.5	9.7	9.9		10.3
i=3	x3(0)=12	11.5	10.7	10.5	10.5	10.5	10.5		10.6
i=4	x4(0)=2	9.75	11.4	11.5	11.5	11.3	11.2		10.9
i=5	x5(0)=3	7.5	8.9	9.5	9.8	10	10.1		10.4
i=6	X6(0)=22	17	15.2	14.2	13.6	13	12.6	…	11.5
avg(t)	10.7	10.7	10.7	10.7	10.7	10.7	10.7	…	10.7

The consensus problem





Robot	State (t=0)	t=1	…	t=10
i=1	x1(0)=5	8.25	…	10.2
i=2	x2(0)=20	10		10.3
i=3	x3(0)=12	11.5		10.6
i=4	x4(0)=2	9.75		10.9
i=5	x5(0)=3	7.5		10.4
i=6	X6(0)=22	17	…	11.5
avg(t)	10.7	10.7	…	10.7

Consensus vs. flooding (tree building + propagation)

Memory storage required?
n increases and.. ?
Switching topology?

The consensus problem

Why is the consensus problem interesting?

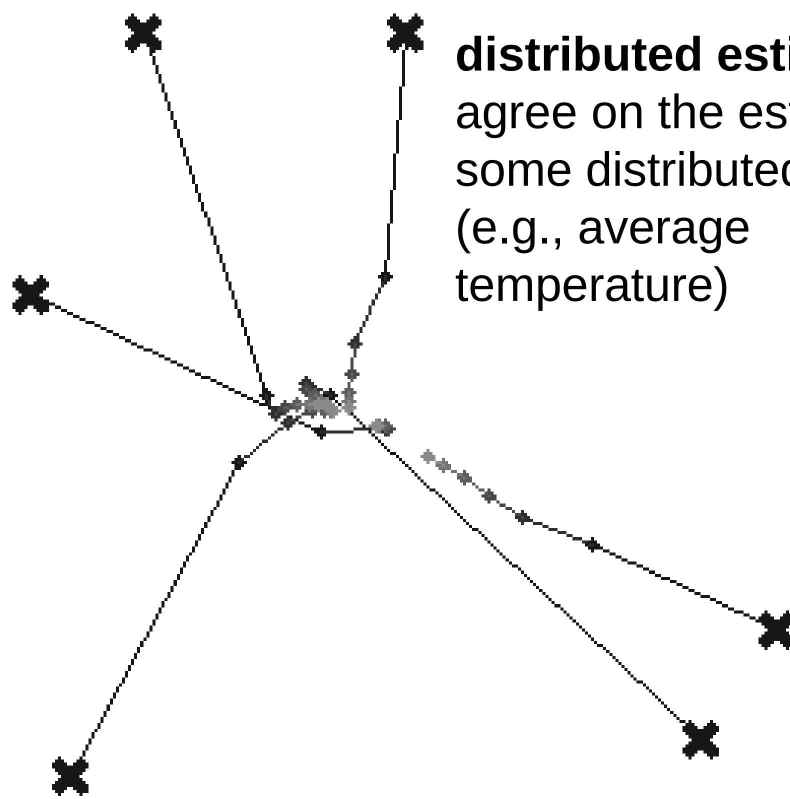
Rendezvous (consensus-based)

meet at a common point (uniform the positions)

Average on x-coordinate
Average on y-coordinate
Robots move to position
(xi(t+1) , yi(t+1))

Rendezvous at the centroid

¿One leader?



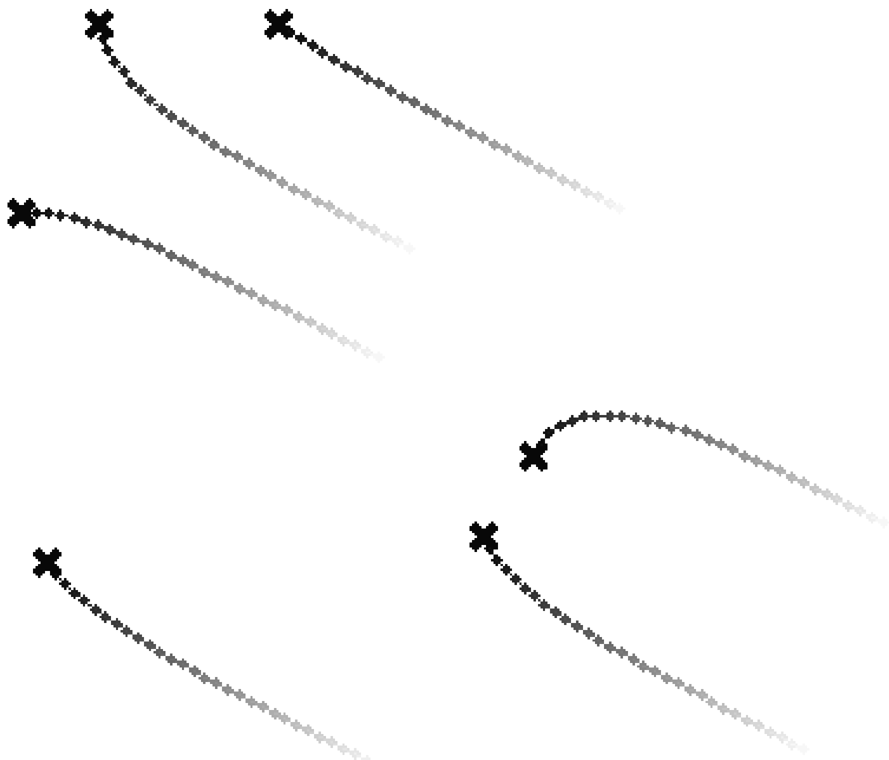
distributed estimation: agree on the estimation of some distributed quantity (e.g., average temperature)

The consensus problem

Why is the consensus problem interesting?

Flocking (consensus-based)

alignment: point in the same direction (uniform the angles)



$$x_i(t+1) = x_i(t) + v_i(t)\Delta t$$

Speed with constant modulus and orientation given by the **averaged orientation**

Alternative: average on speed modulus and on orientation

¿One leader?

T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Phys. Rev. Lett.*, vol. 75, pp. 1226–1229, 1995

The consensus problem Why is the consensus problem interesting?

Circuit pursuit | Orbit motions | Deployment on a ring | Target enclosing

Formation control

E.g., linear (naïve) consensus-based version

Kaveh Fathian, Sleiman Safaoui, Tyler Summers, Nicholas Gans
University of Texas at Dallas

Rewritten

$$\boldsymbol{x_i}\ (\boldsymbol{k}+\mathbf{1}) = \boldsymbol{x_i}\ (\boldsymbol{k}) + \sum_{j\in N_i} \boldsymbol{W_{ij}}(\boldsymbol{x_j}(\boldsymbol{k}) - \boldsymbol{x_i}(\boldsymbol{k}))$$

Now… to keep a fixed relative position between neighbors rij

$$\boldsymbol{x_i}\ (\boldsymbol{k}+\mathbf{1}) = \boldsymbol{x_i}\ (\boldsymbol{k}) + \sum_{j\in N_i} \boldsymbol{W_{ij}}(\boldsymbol{x_j}(\boldsymbol{k}) - \boldsymbol{x_i}(\boldsymbol{k}) - \boldsymbol{r_{ij}})$$

Why it works? x_i(k) remains constant only if the desired relative positions are kept

Equivalently..

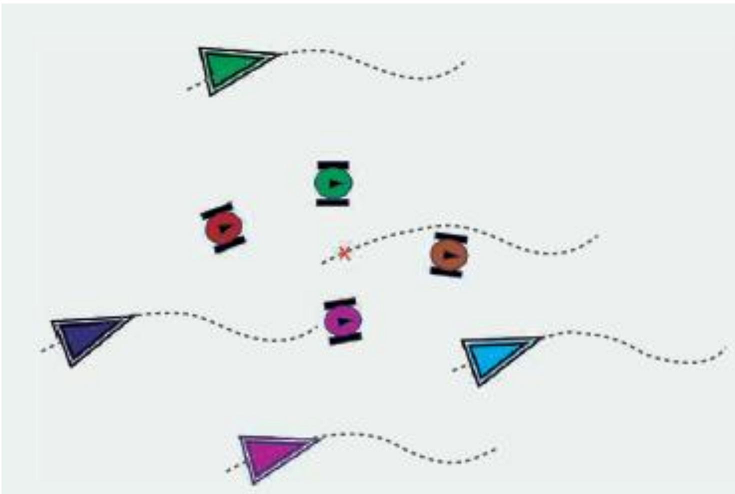
$$\boldsymbol{x_i}\ (\boldsymbol{k}+\mathbf{1}) = \boldsymbol{x_i}\ (\boldsymbol{k}) + \sum_{j\in N_i} \boldsymbol{W_{ij}}(\boldsymbol{x_j}(\boldsymbol{k}) - \boldsymbol{x_i}(\boldsymbol{k})) + \boldsymbol{r_i} \qquad \boldsymbol{r_i} = -\sum_{j\in N_i} \boldsymbol{W_{ij}}\boldsymbol{r_{ij}}$$

Olfati-Saber, R., Fax, J. A., & Murray, R. M. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, *95*(1), 215-233.

The consensus problem

Dynamic consensus

- To **track** the average of references that **change** along time $\boldsymbol{r_i}(\mathrm{k})$



$$\boldsymbol{r_{avg}}(\mathrm{k}) = \frac{\boldsymbol{r_1}(\mathrm{k}) + \cdots + \boldsymbol{r_N}(\mathrm{k})}{N}$$

- Each i robot measures **r_i(k)**. It initializes its state: $\boldsymbol{x_i}(0) = \boldsymbol{r_i}(0)$

$$\boldsymbol{x_i}(\mathrm{k}) = \boldsymbol{r_i}(\mathrm{k}) - \boldsymbol{r_i}(\mathrm{k}-1) +$$

Reference increment

$$+\boldsymbol{x_i}(\mathrm{k}-1) - \alpha \sum_{j \epsilon N_i} \left(\boldsymbol{x_i}(\mathrm{k}-1) - \boldsymbol{x_j}(\mathrm{k}-1) \right)$$

Consensus

Kia, S. S., Van Scoy, B., Cortes, J., Freeman, R. A., Lynch, K. M., & Martinez, S. (2019). Tutorial on dynamic average consensus: The problem, its applications, and the algorithms. IEEE Control Systems Magazine, 39(3), 40-72.

Dynamic consensus

- **Convergence in undirected connected graphs**
- Conditions:
  - Parameter properly selected $\alpha \in \left(0, \frac{1}{d_{max}}\right)$
  - The reference increments are bounded

$$\mathbf{r}_{\mathbf{max}} = \max_{k} \left\| (\mathbf{I} - \mathbf{1}\,\mathbf{1}^T/n)(\mathbf{r}(k) - \mathbf{r}(k+1)) \right\|_2 \text{ finite}$$

- Then, the states converge to a **neighborhood** of the average of the references $\boldsymbol{r}_{\boldsymbol{avg}}(k)$

$$\lim_{k\to\infty} \left| x_i(k) - r_{avg}(k) \right| \le \frac{r_{max}}{\alpha \lambda_2}$$

(Framework)
Algebraic connectivity

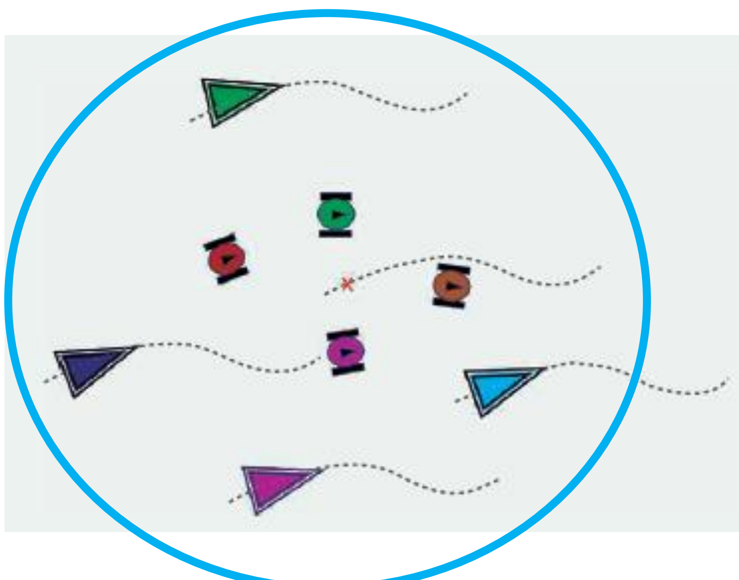
Dynamic consensus

- **Convergence in undirected connected graphs**

$$\alpha \in \left(0, \frac{1}{d_{max}}\right) \qquad \mathbf{r}_{\mathbf{max}} = \max_{k} \left\| (\mathbf{I} - \mathbf{1}\,\mathbf{1}^{T}/n)(\mathbf{r}(k) - \mathbf{r}(k+1)) \right\|_2 \text{ finite}$$

- The states converge to a **neighborhood** of $r_{avg}(k)$

$$\lim_{k\to\infty} \left| x_i(k) - r_{avg}(k) \right| \leq \frac{r_{max}}{\alpha\,\lambda_2}$$



More communication links ?

$\lambda_2 \uparrow$ (Increase connectivity)

Neighborhood shrinks (more accurate)

The consensus problem

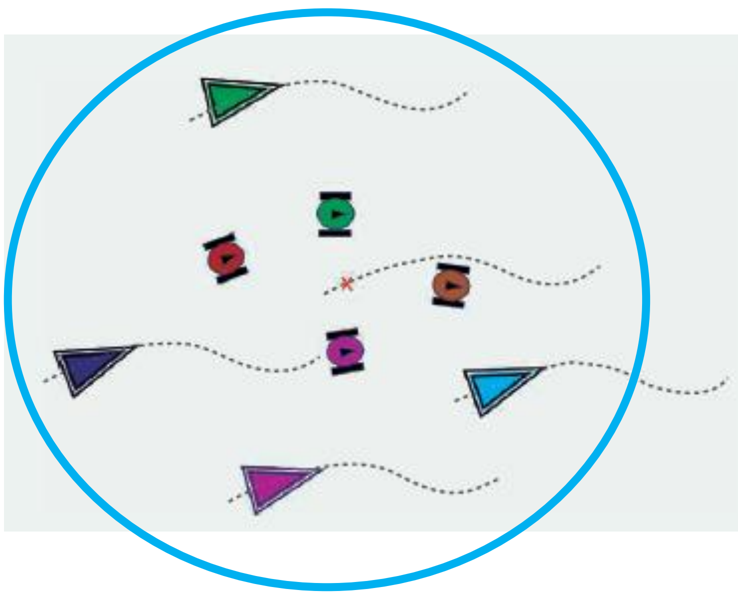
Dynamic consensus

- **Convergence in undirected connected graphs**

$$\alpha \in \left(0, \frac{1}{d_{max}}\right) \qquad \mathbf{r}_{\mathbf{max}} = \max_{k} \left\| (\mathbf{I} - \mathbf{1}\,\mathbf{1}^T/n)(\mathbf{r}(k) - \mathbf{r}(k+1)) \right\|_2 \text{ finite}$$

- The states converge to a **neighborhood** of $r_{avg}(k)$

$$\lim_{k\to\infty} \left| x_i(k) - r_{avg}(k) \right| \leq \frac{r_{max}}{\alpha\, \lambda_2}$$



Any other alternatives to shrink the neighborhood?

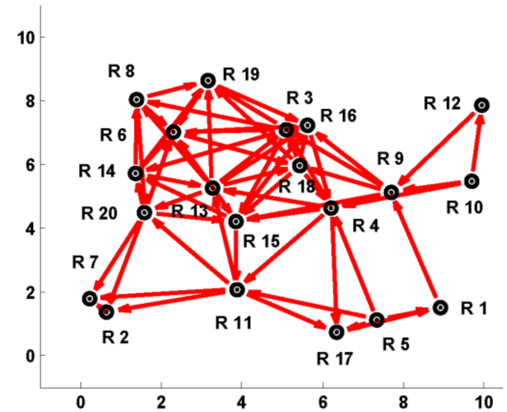
α ↑ (there is a limit !)

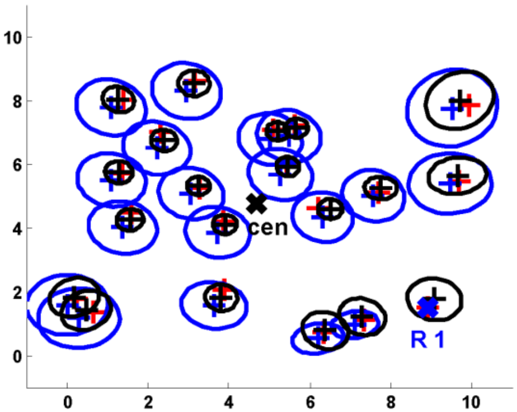
Update more frequently (reference increments between k and k+1 smaller)

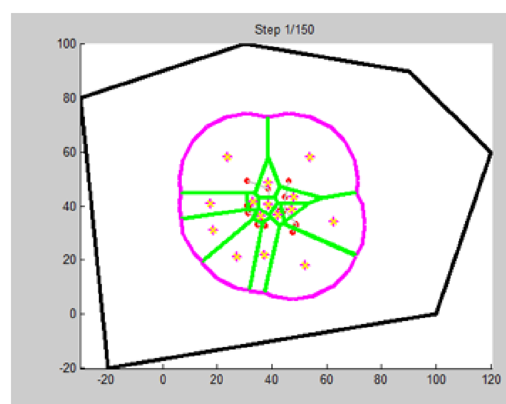
The consensus problem

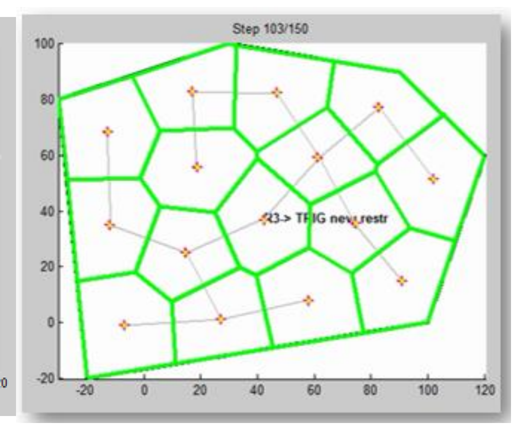
Classical distributed robotic tasks

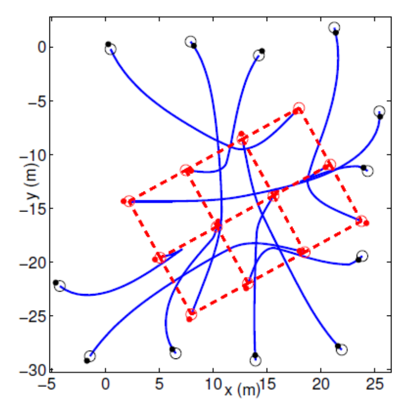
- ***** Agreement and consensus *****
- ***Formation control***
- ***Distributed localization***
- ***Coverage and deployment***
- ***Cooperative transportation***
- ***Rendezvous, swarm aggregation***
- ***Leader-follower tracking***
- ***Containment control***
- ***Connectivity maintenance***
- ***…***











Organización

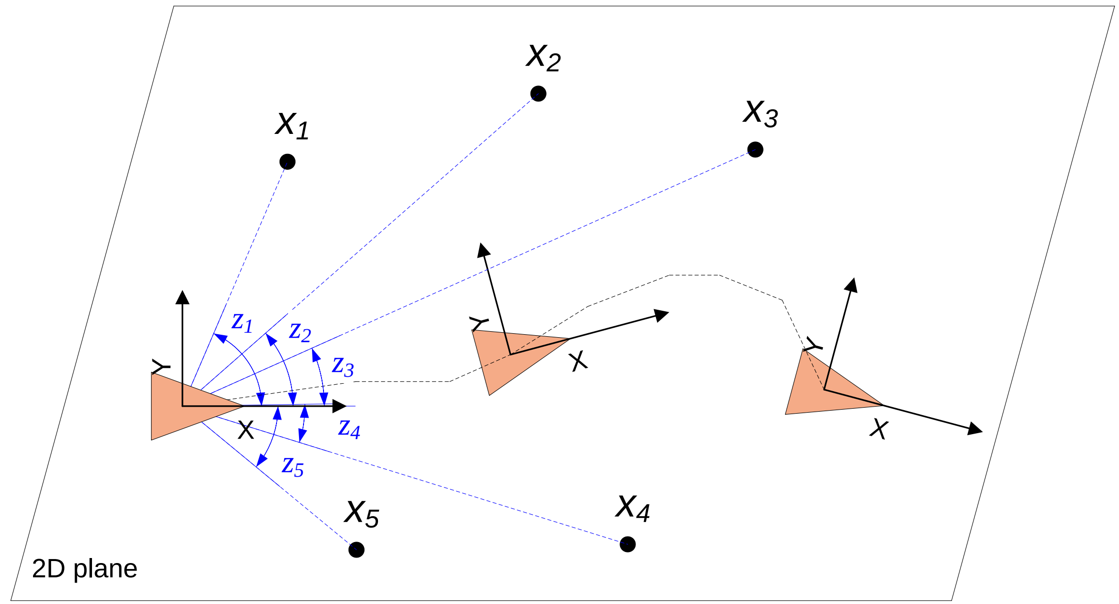
- ❑ Introduction
- ❑ The consensus problem
- ❑ **Dynamic consensus for map merging**
- ❑ Dynamic consensus for multi-leader formation control
- ❑ Consensus for intermittent connectivity
- ❑ Conclusions

Dynamic consensus for map merging

Static/Dynamic Map Merging of Feature-based Stochastic Maps

Feature-based map

- Mean (E[x])
- Covariance matrix (E[(x-x*) (x-x*)^T])



- ✓ (Static / dynamic) (Fixed / time-varying graphs)
- ✓ Convergence conditions, convergence speed
- ✓ Unbiased mean, consistent estimates
- ✓ Improvements to decrease the exchanged data

[R. Aragues, C. Sagues (2008). Parameterization and initialization of bearing-only information: a discussion. In Proc. Fifth International Conference on Informatics in Control, Automation and Robotics, ICINCO 2008]

Dynamic consensus for map merging

Static Map Merging

(assuming the common reference frame and the data association are solved)

- Local map of each robot ***i***:
  - mean $\hat{\mathbf{x}}_i \in \mathbb{R}^{\mathcal{M}_i}$
  - covariance matrix $\Sigma_i \in \mathbb{R}^{\mathcal{M}_i \times \mathcal{M}_i}$
  - size of the local map $\mathcal{M}_i = szr + m_i \; szf$

- Label vector of robot ***i*** $L_i = (L_1^i, \ldots, L_{m_i}^i)$
  - Label $L_r^i = (a_r^i, b_r^i)$ associated to a feature f_r^i
  - Labels sorted in lexicographical order

$\hat{\mathbf{x}}_i \in \mathbb{R}^{\mathcal{M}_i}$ $\Sigma_i \in \mathbb{R}^{\mathcal{M}_i \times \mathcal{M}_i}$ arranged according to L_i

$f_r^i = f_s^j$ (and will be fused together) iff $a_r^i = a_s^j$ and $b_r^i = b_s^j$

$$L_r^i = (a_r^i, b_r^i) \qquad L_s^j = (a_s^j, b_s^j)$$

Dynamic consensus for map merging

Static Map Merging

(assuming the common reference frame and the data association are solved)

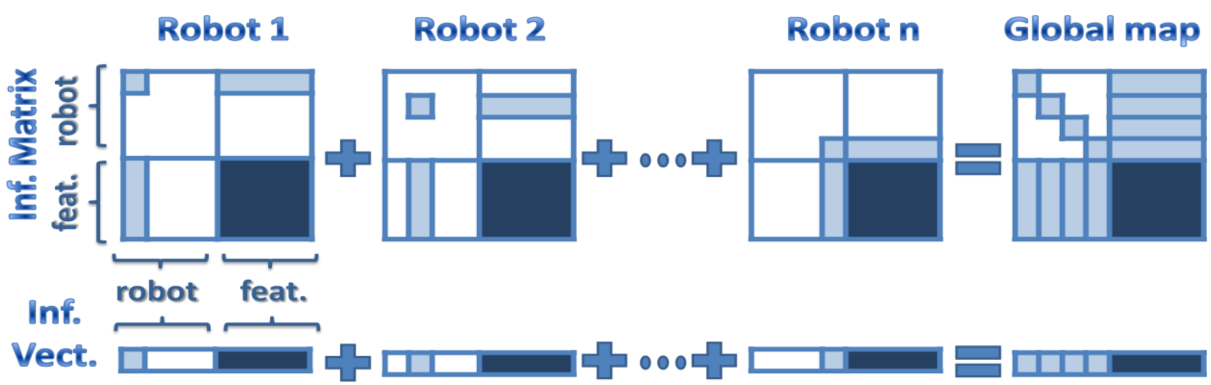
Local maps (initial data): observations of the true feature and robot positions $\mathbf{x} \in \mathbb{R}^{\mathcal{M}_G}$

$$\hat{\mathbf{x}}_i = H_i \mathbf{x} + H_i \mathbf{v}_i$$

$$H_i \in \mathbb{R}^{\mathcal{M}_i \times \mathcal{M}_G} \qquad \mathbf{v}_i \sim N(0, H_i^T \Sigma_i H_i) \qquad \mathcal{M}_G = n\ szr + m\ szf$$

Goal global map (IF, mean, covariance) to be computed in a distributed way:

$$I_G = \sum_{i=1}^{n} H_i^T \Sigma_i^{-1} H_i, \qquad \mathbf{i}_G = \sum_{i=1}^{n} H_i^T \Sigma_i^{-1} \hat{\mathbf{x}}_i, \qquad \hat{\mathbf{x}}_G = (I_G)^{-1} \mathbf{i}_G \qquad \Sigma_G = (I_G)^{-1}$$



Dynamic consensus for map merging

Static Map Merging

(assuming the common reference frame and the data association are solved)

Initialization: $L_i(0) = L_i, \; I_G^i(0) = \Sigma_i^{-1}, \; \mathbf{i}_G^i(0) = \Sigma_i^{-1}\hat{\mathbf{x}}_i.$

Algorithm (iterations):

$$I_G^i(t+1) = \sum_{j=1}^{n} W_{i,j} (H_{j,t}^{i,t+1})^T I_G^j(t) H_{j,t}^{i,t+1},$$

$$\mathbf{i}_G^i(t+1) = \sum_{j=1}^{n} W_{i,j} (H_{j,t}^{i,t+1})^T \mathbf{i}_G^j(t),$$

With label update $L_j(t), \; j \in \mathcal{N}_i \cup \{i\}, \longrightarrow L_i(t+1)$

Metropolis weights

$$W_{i,j} = \begin{cases} \frac{1}{1+\max\{|\mathcal{N}_i|,|\mathcal{N}_j|\}} & if \; j \in \mathcal{N}_i, j \neq i \\ 0 & if \; j \notin \mathcal{N}_i, j \neq i \\ 1 - \sum_{j \in \mathcal{N}_i} W_{i,j} & if \; i = j \end{cases} .$$

Dynamic consensus for map merging

Static Map Merging

(assuming the common reference frame and the data association are solved)

Properties

- Convergence for *fixed* and *jointly connected* graphs to the goal global map. Speed depending on the algebraic connectivity (discussed later)

$$|[I_G^i(t)]_{r,s} - [I_G]_{r,s}| \leq (\gamma)^t \sqrt{n} \max_j \left\{ \left| [I_G^j(0)]_{r,s} - [I_G]_{r,s} \right| \right\}$$

$$\gamma = |\lambda_2(W)|$$

- Unbiased mean

$$\hat{\mathbf{x}}_G^i(t) = \left(I_G^i(t)\right)^{-1} \mathbf{i}_G^i(t) \qquad \mathbf{E}\left[\hat{\mathbf{x}}_G^i(t)\right] = \mathbf{x}$$

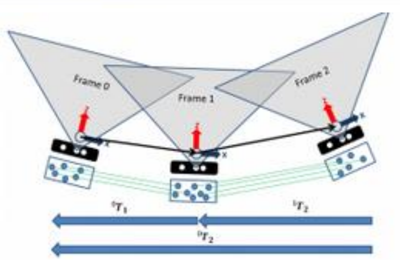
- Consistent covariance

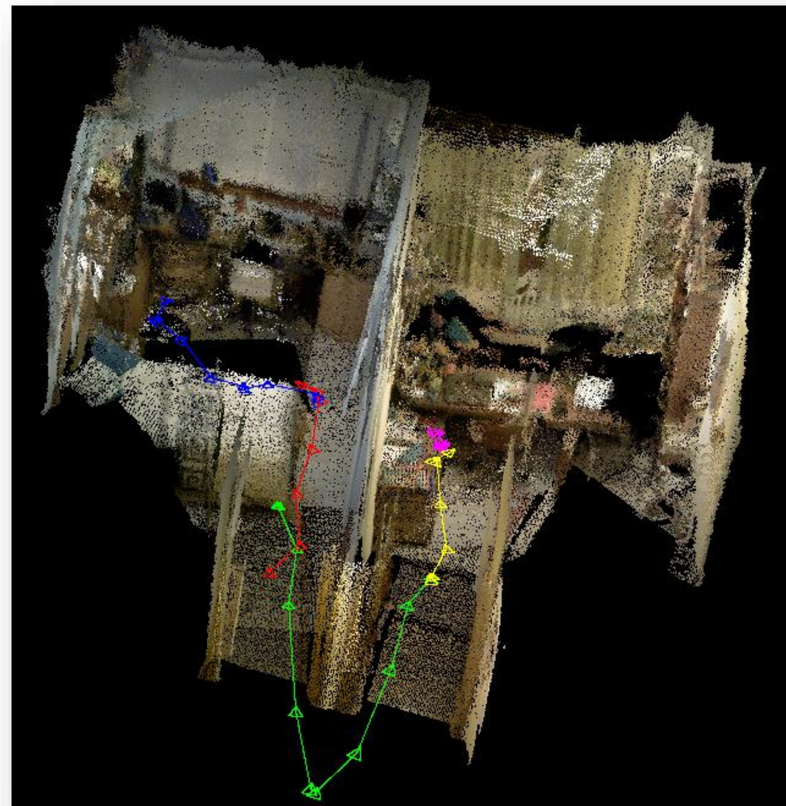
$$\mathbf{E}\left[\left(\hat{\mathbf{x}}_G^i(t) - \mathbf{x}\right)\left(\hat{\mathbf{x}}_G^i(t) - \mathbf{x}\right)^T\right] \preceq \left(\hat{I}_G^i(t)\right)^{-1}$$

Dynamic consensus for map merging

Static Map Merging (assuming common frame and data association are solved)

Experiments with RGB-D data



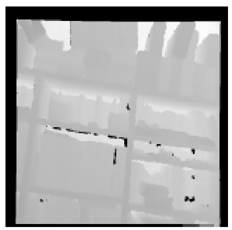


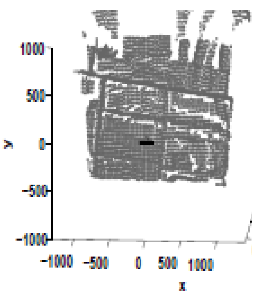
[S.Ayuso, C.Sagues, R.Aragues. Distributed Localization and Scene Reconstruction from RGB-D data. International Conference on Informatics in Control Automation and Robotics, pp. 377 – 384, Reykjavik, Iceland, July 2013]

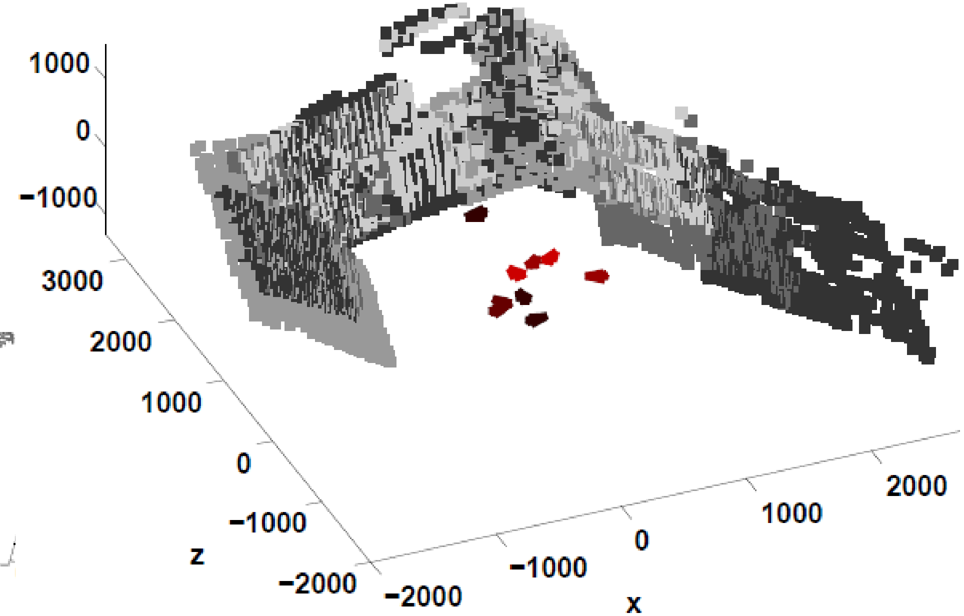
Dynamic consensus for map merging

Dynamic Map Merging

The local maps change as robots move







[R. Aragues, J. Cortes, C. Sagues. Dynamic Consensus for Merging Visual Maps under Limited Communications. IEEE Int. Conf. on Robotics and Automation, Anchorage AK, USA, 3032-3037, May 2010]

[R. Aragues, J. Cortes, C. Sagues. Distributed Consensus on Robot Networks for Dynamically Merging Feature-Based Maps. IEEE Transactions on Robotics, 28(4):840-854, 2012]

[R. Aragues, C. Sagues, Y. Mezouar. Feature-Based Map Merging with Dynamic Consensus on Information Increments. IEEE Int. Conf. on Robotics and Automation, Karlsruhe, Germany, 2710 – 2715, May 2013]

[R. Aragues, C. Sagues, Y. Mezouar. Feature–based Map Merging with Dynamic Consensus on Information Increments. Autonomous Robots 38 (3): 243–259, 2015]

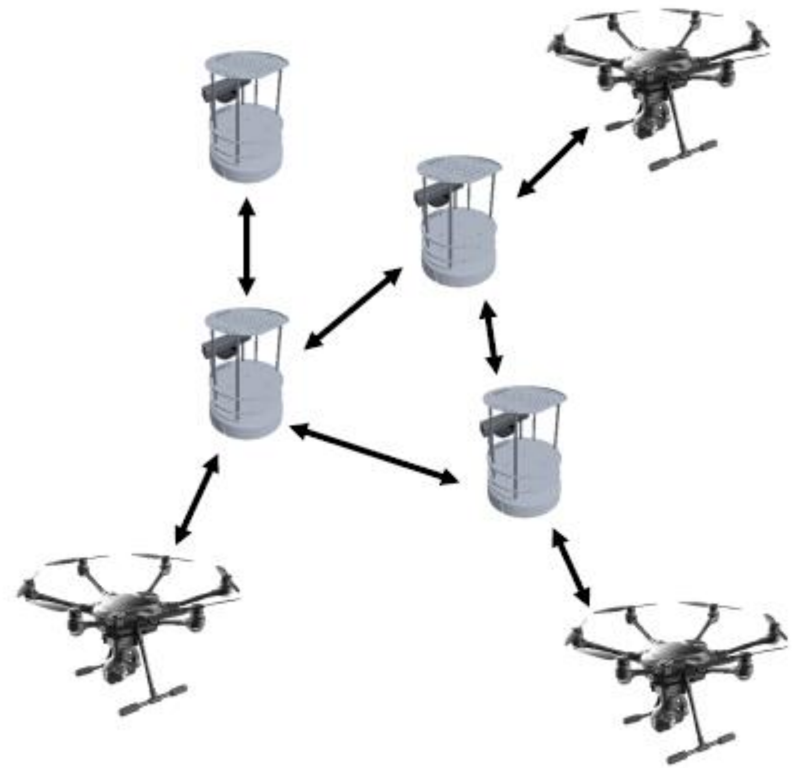
[R. Aragues, C. Sagues, Y. Mezouar. Parallel and Distributed Map Merging and Localization: Algorithms, Tools and Strategies for Robotic Networks. Springer – ISBN 978-3-319-25884-3, 2015]

Organización

Dynamic consensus for multi-leader formation control

Containment robot formation

- Some robots have an advantage: top view, better sensors, etc.
- Other robots need to be directed: e.g., constrained resources
- The robots communicate
- Leader robots may construct safe pathways for follower robots



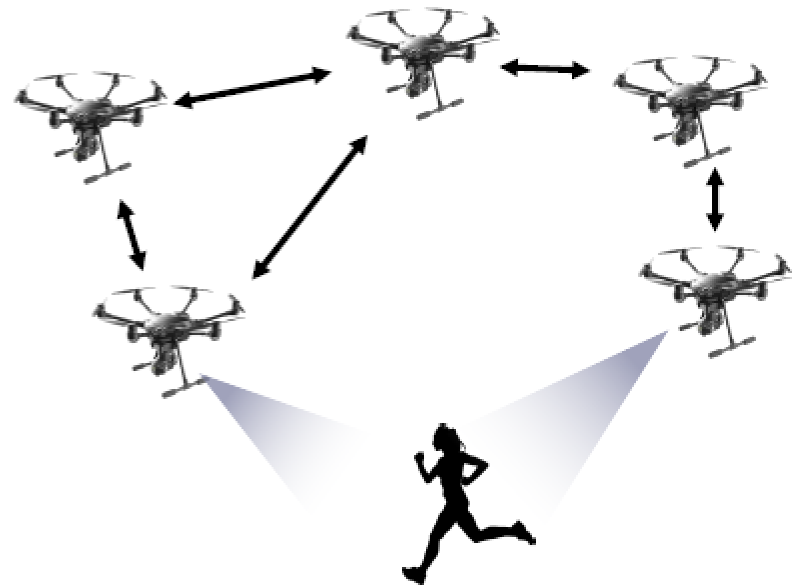
- Goal: compute the geometric center of the leaders
- Then, make a formation around this trajectory

R. Aldana-López, D. Gómez-Gutiérrez, R. Aragüés and C. Sagüés, "Dynamic Consensus With Prescribed Convergence Time for Multileader Formation Tracking," in *IEEE Control Systems Letters*, vol. 6, pp. 3014-3019, 2022, doi: 10.1109/LCSYS.2022.3181784.

Dynamic consensus for multi-leader formation control

Cooperative target estimation and tracking

- A team of robots aim to detect and track a moving target
- The robots communicate
- Only some of them have direct line of sight, with imperfect measurements



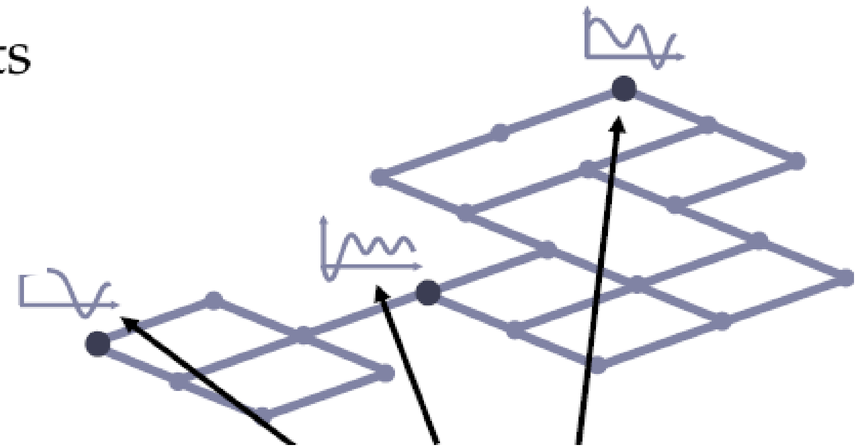
Goal: share information to obtain a single global estimate of the target accross the network.

Then, make a formation around this averaged estimate

R. Aldana-López, D. Gómez-Gutiérrez, R. Aragüés and C. Sagüés, "Dynamic Consensus With Prescribed Convergence Time for Multileader Formation Tracking," in *IEEE Control Systems Letters*, vol. 6, pp. 3014-3019, 2022, doi: 10.1109/LCSYS.2022.3181784.

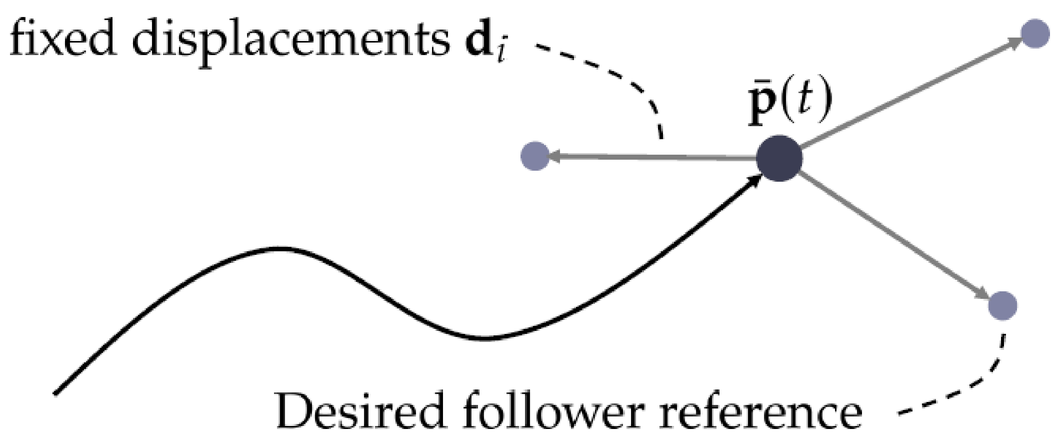
Dynamic consensus for multi-leader formation control

- There is a network of N agents
- N_L of them are leaders
- The rest are followers



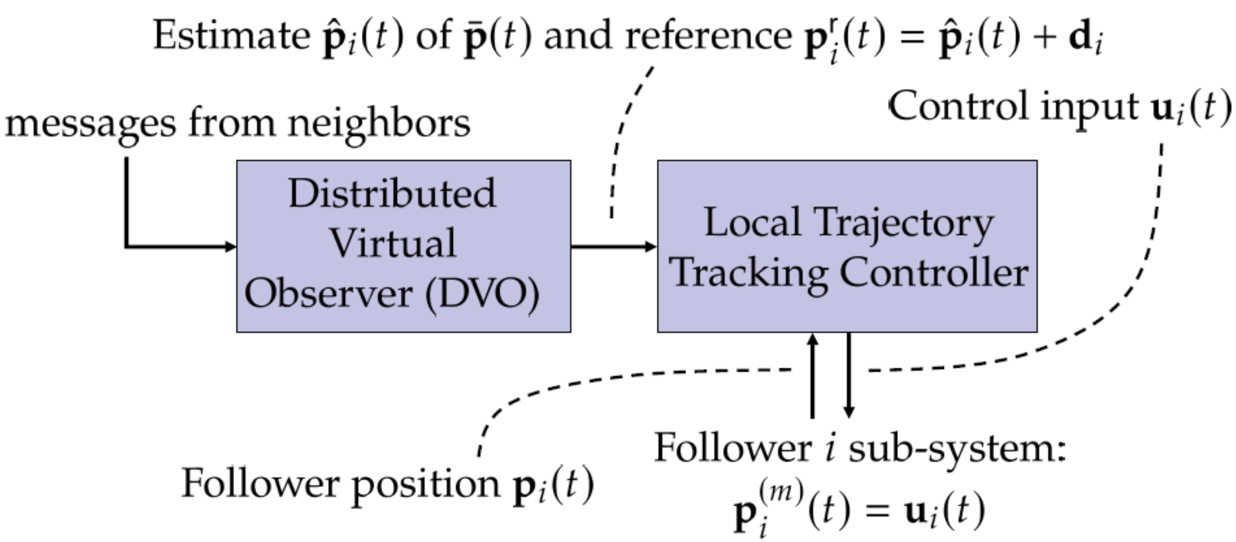
- Leaders provide time-varying "signals" $\mathbf{p}_1(t), \ldots, \mathbf{p}_{N_L}(t)$

Goal: compute $\bar{\mathbf{p}}(t) := \dfrac{\mathbf{p}_1(t) + \cdots + \mathbf{p}_{N_L}(t)}{N_L}$
at each agent and achieve a formation around it.

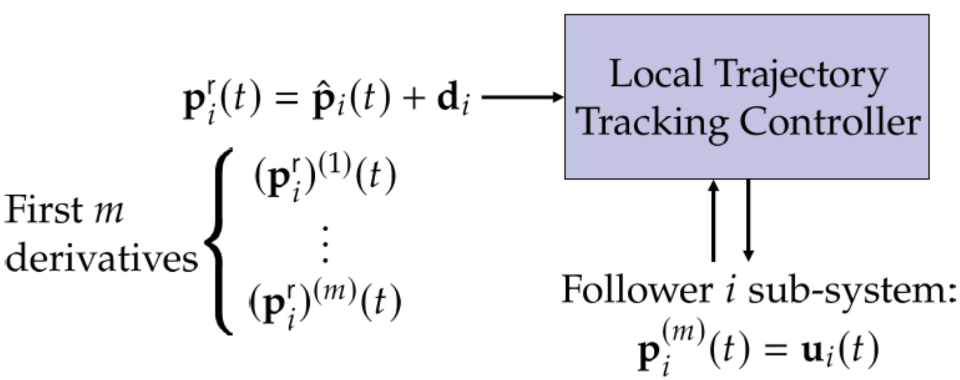


Dynamic consensus for multi-leader formation control

Two-step solution at a follower agent i



If the sub-system is or relative degree $m > 1$ then we need more information...

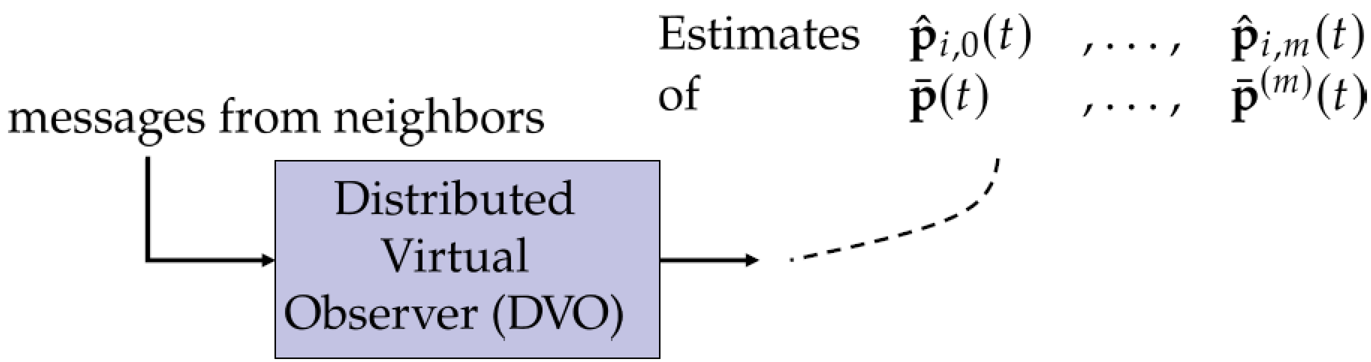


For example, a linear controller

$$\begin{aligned}\mathbf{u}_i(t) = \; & \rho_0(\mathbf{p}_i(t) - \mathbf{p}_i^{\mathrm{r}}(t)) + \\ & \vdots \\ & + \rho_{m-1}(\mathbf{p}_i^{(m-1)}(t) - (\mathbf{p}_i^{\mathrm{r}})^{(m-1)}(t)) \\ & + (\mathbf{p}_i^{\mathrm{r}})^{(\mu)}(t)\end{aligned}$$

Dynamic consensus for multi-leader formation control

Two-step solution at a follower agent i



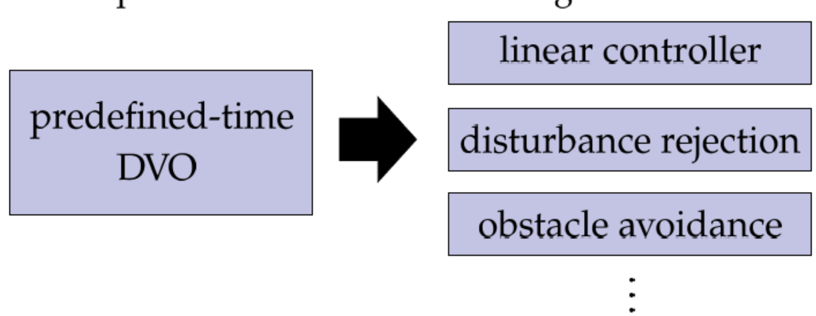
We want "real-time" performance:

$$\hat{\mathbf{p}}_{i,\mu}(t) = \bar{\mathbf{p}}^{(\mu)}(t) \text{ for all } t \geq T_c$$

with user predefined convergence time T_c

A predefined convergence time allows:

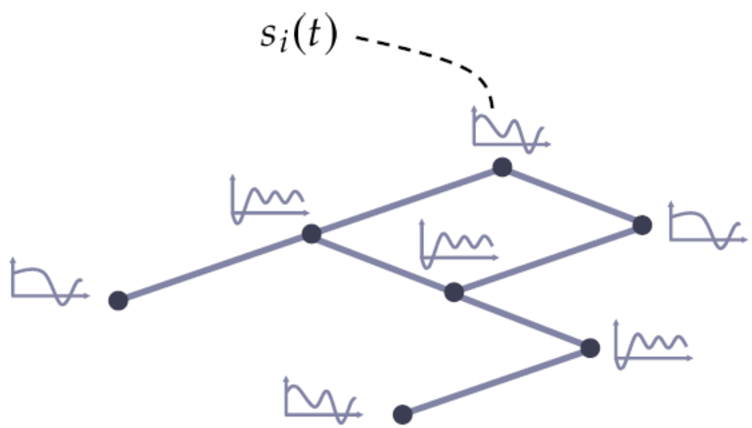
- application specific real-time constraints
- trivial observer-controller separation principle
  $\Longrightarrow$ decoupled observer-controller design



Dynamic consensus for multi-leader formation control

Exact dynamic consensus | Modulating functions | Ratio consensus

Exact Dynamic Consensus (EDC)



- N agents
- Network modeled with unndirected graph $\mathcal{G}$
- Agent i as access to local signal $s_i(t)$

EDC algorithms compute $\bar{s}(t) = \dfrac{s_1(t) + \cdots + s_\mathsf{N}(t)}{\mathsf{N}}$

as well as $\bar{s}^{(1)}(t), \ldots, \bar{s}^{(m)}(t)$

Dynamic consensus for multi-leader formation control

Exact Dynamic Consensus of High Order (EDCHO)

The EDCHO algorithm is composed by (at the i-th agent)

- m Internal state variables $x_{i,0}(t), \ldots, x_{i,m}(t)$
- m outputs $y_{i,0}(t), \ldots, y_{i,m}(t)$ which estimate $\bar{s}(t), \ldots, \bar{s}^{(m)}(t)$ concretely,

$$y_{i,\mu}(t) = s_i^{(\mu)}(t) - x_{i,\mu}(t), \quad \mu \in \{0, \ldots, m\}$$

$$\dot{x}_{i,\mu} = x_{i,\mu+1} + k_\mu \sum_{j \in \mathcal{N}_i} \lceil y_{i,0} - y_{j,0} \rfloor^{\frac{m-\mu}{m+1}} \quad \text{for } \mu \in \{0, \ldots, m-1\}$$

$$\dot{x}_{i,m} = + k_m \sum_{j \in \mathcal{N}_i} \text{sign}(y_{i,0} - y_{j,0})$$

Chain of integrators (first terms); error correction (summation terms)

Only $y_{i,0}(t)$ is shared

$\mathcal{N}_i$:= neighbors of i

$\lceil \bullet \rfloor^\alpha := |\bullet|^\alpha \text{sign}(\bullet)$

Dynamic consensus for multi-leader formation control

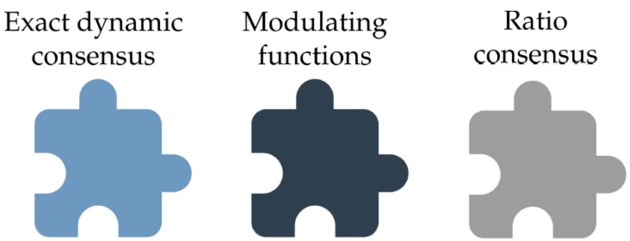
Can we set the convergence time T to a user defined T_c?

- T is a function of initial conditions $\tilde{y}_{i,\mu}(0), \forall i, \mu$
- Such dependence is (still) unknown
- Initial conditions are unknown as well
- **but** we know that if $\tilde{y}_{i,\mu}(0) = 0, \forall i, \mu$ then $T = 0$

Finite time convergence of EDCHO:

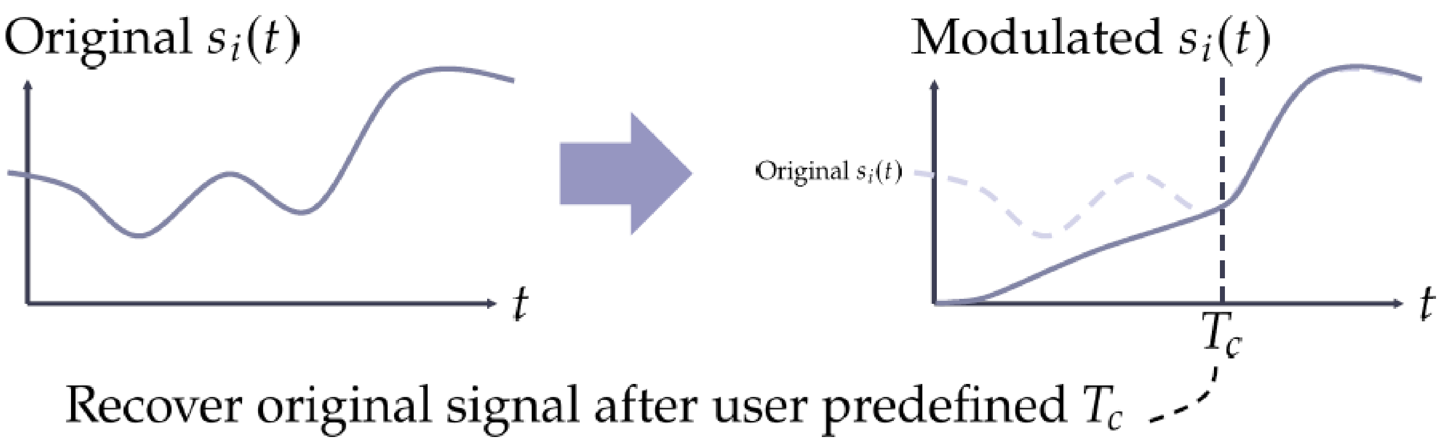
$$y_{i,\mu}(t) = \bar{s}^{(\mu)}(t), \ \forall t \geq T$$

Dynamic consensus for multi-leader formation control



Modulating functions for Predefined-time EDCHO

Idea: Modulate the signals $s_i(t)$ to make sure $\tilde{y}_{i,\mu}(t) = 0$ right from the start



Dynamic consensus for multi-leader formation control

Exact dynamic consensus | Modulating functions | Ratio consensus

Ratio strategy

$N - N_L$ followers signals

N_L leaders position

For $t \geq T_c$: $\hat{\mathbf{p}}_{i,\mu}(t) = \dfrac{\mathbf{p}_1^{(\mu)}(t) + \cdots + \mathbf{p}_{N_L}^{(\mu)}(t) + \mathbf{0} + \cdots + \mathbf{0}}{N}$

$N - N_L$ followers labels

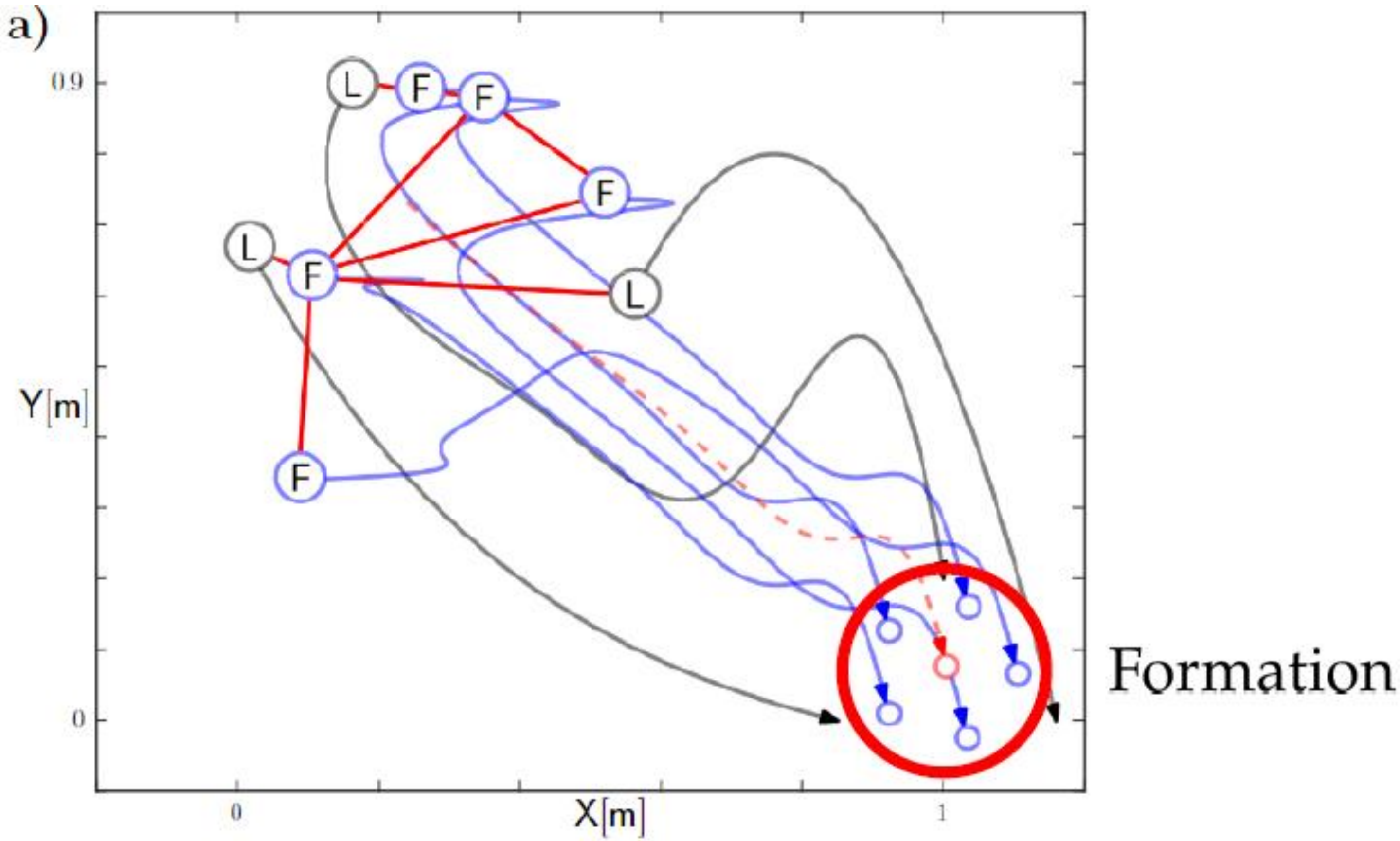
N_L leaders labels

For $t \geq T_c$: $\hat{\ell}_{i,\mu}(t) = \dfrac{1 + \cdots + 1 + 0 + \cdots + 0}{N}$

Combining both outputs...

$$\text{For } t \geq T_c : \quad \frac{\hat{\mathbf{p}}_{i,\mu}(t)}{\hat{\ell}_{i,\mu}(t)} = \frac{\dfrac{\mathbf{p}_1^{(\mu)}(t) + \cdots + \mathbf{p}_{N_L}^{(\mu)}(t)}{N}}{\dfrac{N_L}{N}}$$

Dynamic consensus for multi-leader formation control



Dynamic consensus for multi-leader formation control

R. Aldana-López, D. Gómez-Gutiérrez, R. Aragüés and C. Sagüés, "Dynamic Consensus With Prescribed Convergence Time for Multileader Formation Tracking," in *IEEE Control Systems Letters*, vol. 6, pp. 3014-3019, 2022, doi: 10.1109/LCSYS.2022.3181784.

Rodrigo Aldana-López, Rosario Aragüés, Carlos Sagüés, Perception-latency aware distributed target tracking, Information Fusion, Volume 99, 2023, 101857, ISSN 1566-2535, https://doi.org/10.1016/j.inffus.2023.101857.

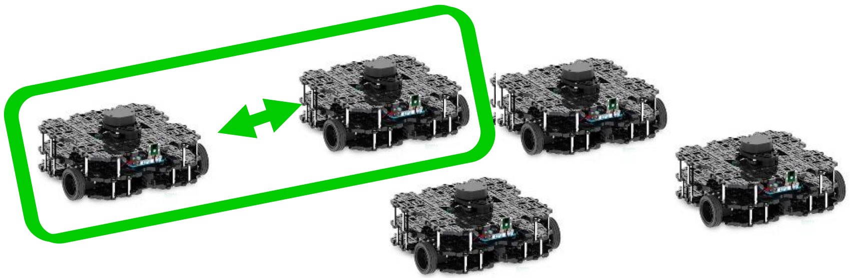
Rodrigo Aldana-López, Rosario Aragüés, Carlos Sagüés, REDCHO: Robust Exact Dynamic Consensus of High Order, Automatica, Volume 141, 2022, 110320, ISSN 0005-1098, https://doi.org/10.1016/j.automatica.2022.110320.

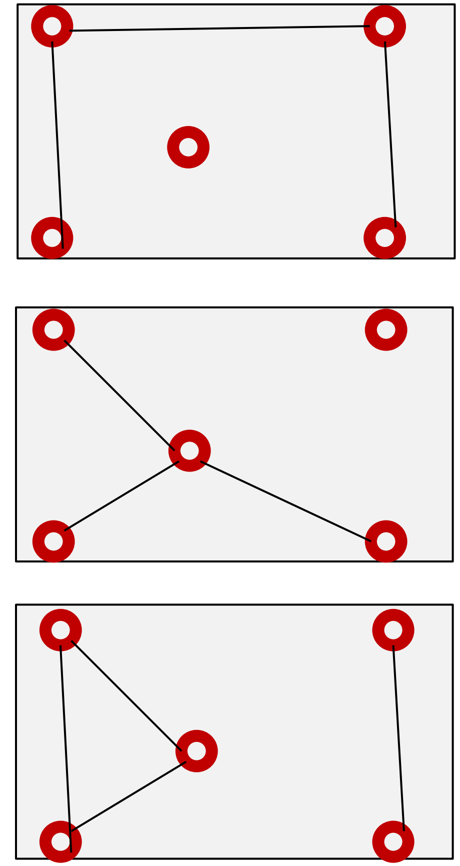
Rodrigo Aldana-López, Rosario Aragüés, Carlos Sagüés, EDCHO: High order exact dynamic consensus, Automatica, Volume 131, 2021, 109750, ISSN 0005-1098, https://doi.org/10.1016/j.automatica.2021.109750.

Organización

- Introduction
- The consensus problem
- Dynamic consensus for map merging
- Dynamic consensus for multi-leader formation control
- **Consensus for intermittent connectivity**
- Conclusions

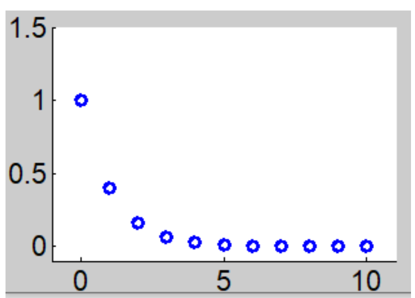
Consensus for intermittent connectivity

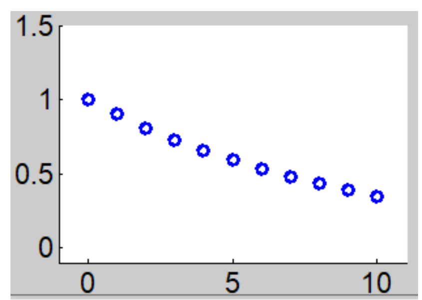




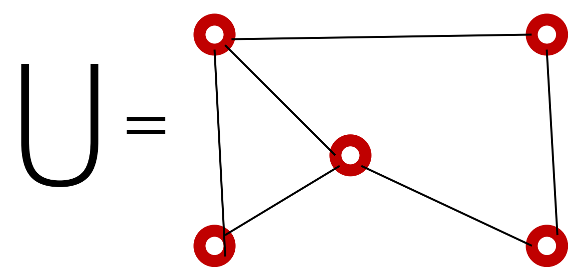
If the union of the graphs that occur *infinitely often* is *jointly connected*

Convergence rate? More, e.g., existence of an interval of *joint connectivity*



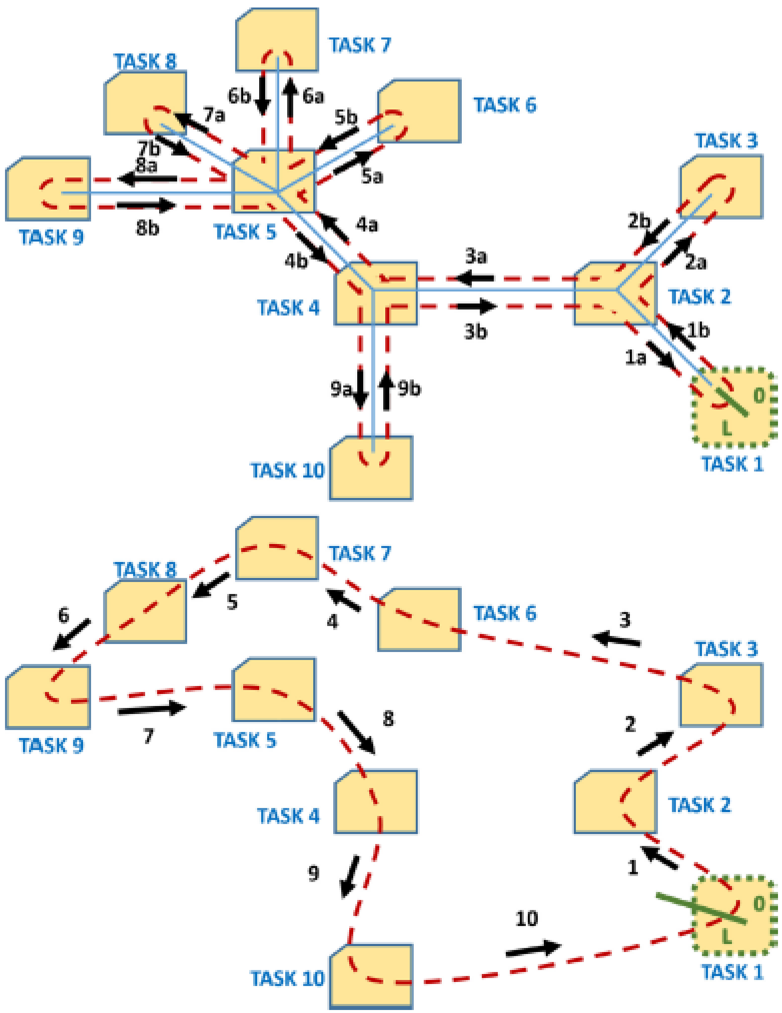


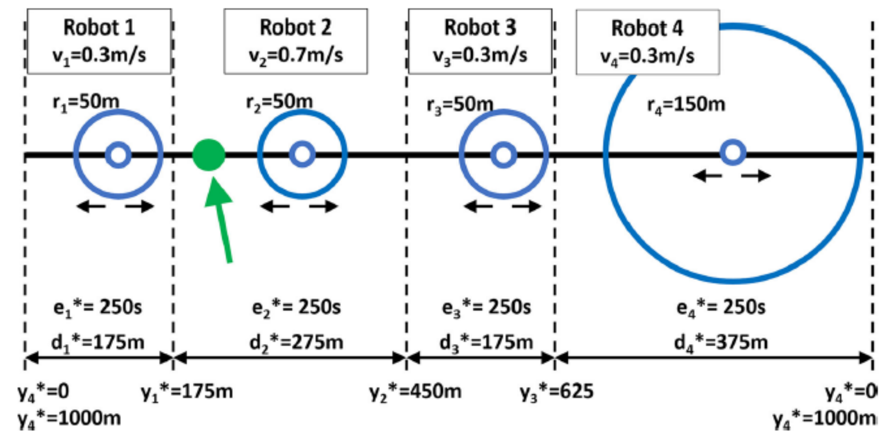
k= 0..10 0.4^k 0.9^k



R. Aragues, D. V. Dimarogonas, P. Guallar and C. Sagues, "Intermittent Connectivity Maintenance With Heterogeneous Robots," in *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 225-245, Feb. 2021, doi: 10.1109/TRO.2020.3014521.

Consensus for intermittent connectivity





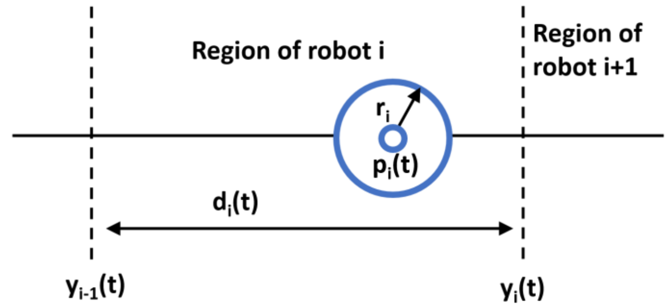
R. Aragues, D. V. Dimarogonas, P. Guallar and C. Sagues, "Intermittent Connectivity Maintenance With Heterogeneous Robots," in *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 225-245, Feb. 2021, doi: 10.1109/TRO.2020.3014521.

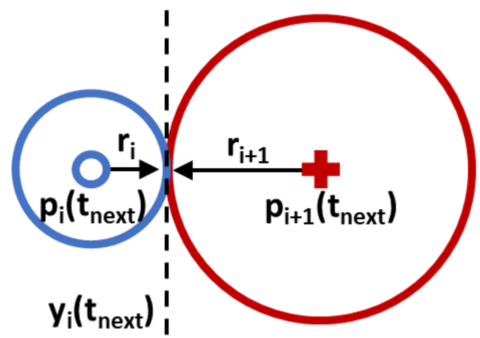
Consensus for intermittent connectivity

Encuentros entre robots

- Actualización de límites yi e yi-1
- Convergencia:
    - Orientaciones balanceadas: Tiempos de espera -> 0
    - Orientaciones desbalanceadas: tiempos de espera en una dirección-> 0

$$y_i(t_e^+) = \frac{v_{i+1}(y_{i-1}(t_e) + 2r_i) + v_i(y_{i+1}(t_e) - 2r_{i+1})}{v_i + v_{i+1}}.$$





R. Aragues, D. V. Dimarogonas, P. Guallar and C. Sagues, "Intermittent Connectivity Maintenance With Heterogeneous Robots," in *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 225-245, Feb. 2021, doi: 10.1109/TRO.2020.3014521.

Consensus for intermittent connectivity

$$t_\star = \frac{d_1^\star - 2r_1}{v_1} = \frac{d_2^\star - 2r_2}{v_2} = \cdots = \frac{d_n^\star - 2r_n}{v_n}$$

$$t_\star = \left(L - 2\sum_{i=1}^{n} r_i\right) / (v_1 + v_2 + \cdots + v_n).$$

Proposition 5.1: (Weighted consensus on traversing times [21, Prop. 5.1]): Assume that Algorithm 3.2 gives rise to a network in which the set of communication graphs that occur infinitely often are jointly connected. Then, the traversing times $e_i(t)$, region lengths $d_i(t)$, and boundaries $y_i(t)$ [(3), (2), (16)] asymptotically converge to the goal values $t_\star$, $d_i^\star$, $y_i^\star$ in (7), (8), and (9), for $i = 1, \ldots, n$.

$$e_i(t_e^+) = e_i(t_e) + \frac{\epsilon_i}{v_i}(e_{i+1}(t_e) - e_i(t_e)),$$

$$e_{i+1}(t_e^+) = e_{i+1}(t_e) - \frac{\epsilon_i}{v_{i+1}}(e_{i+1}(t_e) - e_i(t_e)).$$

$$\epsilon_i = \frac{v_i v_{i+1}}{v_i + v_{i+1}}$$

$$\lim_{t\to\infty} e_i(t) = \frac{\sum_{j=1}^n v_j e_j(0)}{v_1 + \cdots + v_n} = \frac{\sum_{j=1}^n \frac{v_j(d_j(0) - 2r_j)}{v_j}}{v_1 + \cdots + v_n} = t_\star$$

[R. Aragues and D. V. Dimarogonas. *Intermittent connectivity maintenance with heterogeneous robots using a beads-on-a-ring strategy*. American Control Conference (ACC), pp. 120–126, 2019]

R. Aragues, D. V. Dimarogonas, P. Guallar and C. Sagues, "Intermittent Connectivity Maintenance With Heterogeneous Robots," in *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 225-245, Feb. 2021, doi: 10.1109/TRO.2020.3014521.

Consensus for intermittent connectivity

[R. Aragues and D. V. Dimarogonas. *Intermittent connectivity maintenance with heterogeneous robots using a beads-on-a-ring strategy*. American Control Conference (ACC), pp. 120–126, 2019]

R. Aragues, D. V. Dimarogonas, P. Guallar and C. Sagues, "Intermittent Connectivity Maintenance With Heterogeneous Robots," in *IEEE Transactions on Robotics*, vol. 37, no. 1, pp. 225-245, Feb. 2021, doi: 10.1109/TRO.2020.3014521.

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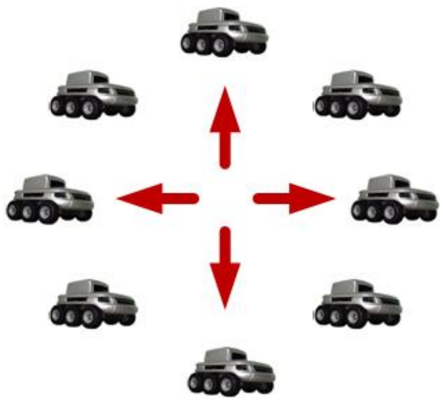
- **Conclusions**

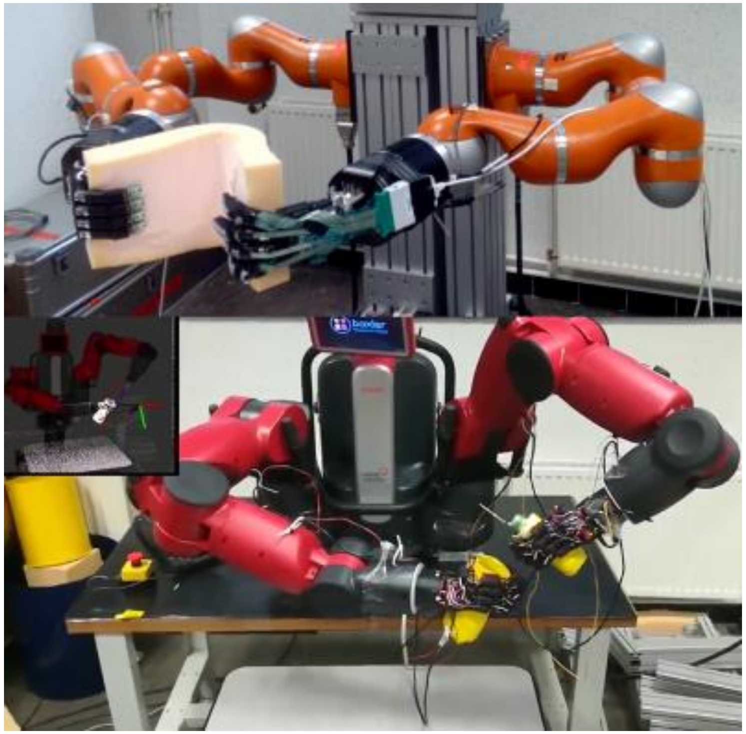
Conclusions

- The consensus problem
- Connections with several distributed problems
- Agree on one or more elements => emerging behaviors
- Scalable
- Distributed
- Robust
- Several communication graphs
- Asymptotic / Finite-time



Conclusions

Proyecto COMMANDIA (Collaborative Robotic Mobile Manipulation of Deformable Objects in Industrial Applications) SOE2/P1/F0638 Interreg Sudoe



http://commandia.unizar.es/es/lo-basico-de-commandia/

sigma CLERMONT

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Departamento de Informática e Ingeniería de Sistemas Universidad Zaragoza

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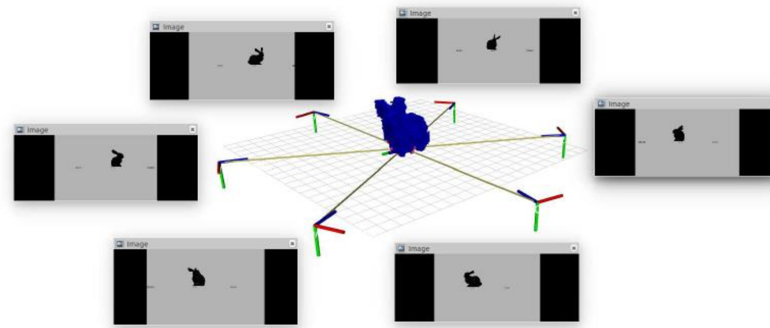
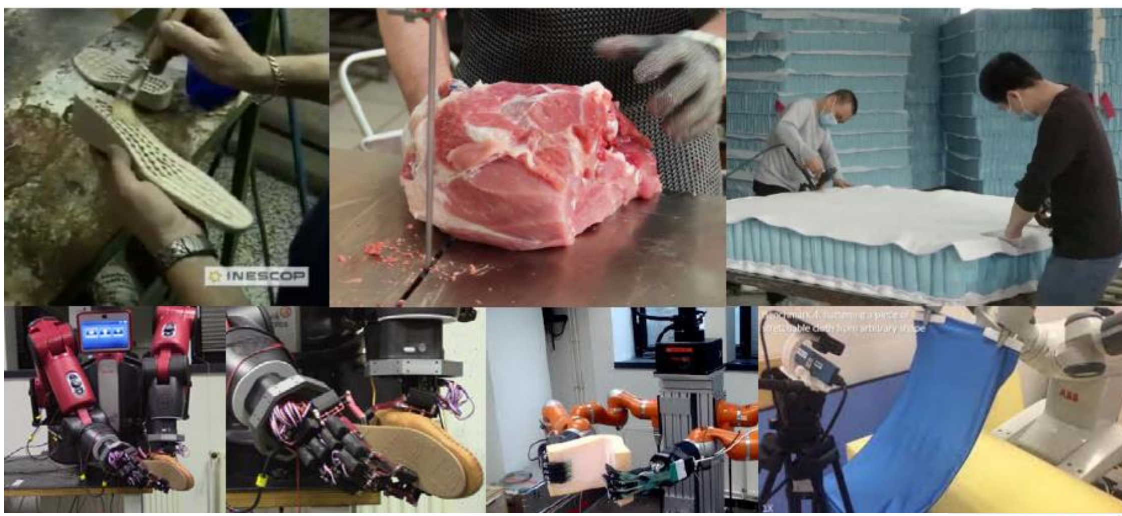
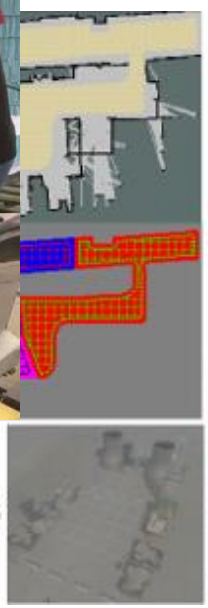
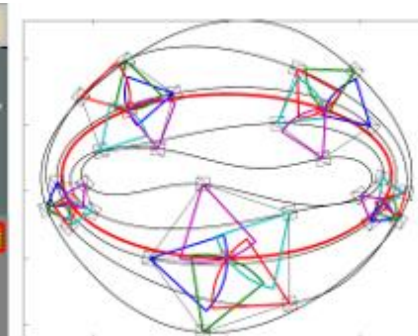
Conclusions

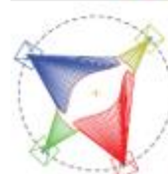
Proyecto DEFORMS (Deformation control of Flexible Objects with cooperative Robots in Manufacturing Sectors) TED2021-130224B-I00

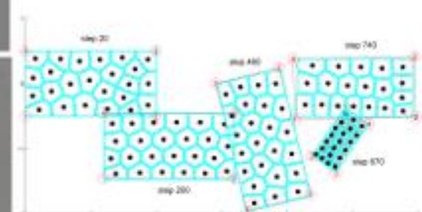












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Conclusions

Proyecto CCOUNTRYBOTS (Cooperative robots for monitorization and deformable goods transport in the countryside) PID2021-124137OB-I00

- Monitorización de cultivos (entornos relativamente estáticos)
- Monitorización de ganado (entornos altamente dinámicos)
- Percepción cooperativa
- Consenso sobre la información obtenida
- Coordinación, comunicación y control
- Mitigación del efecto de los retrasos en las comunicaciones

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Distributed dynamic consensus in multi-robot systems

Rosario ARAGUES

raragues@unizar.es

Universidad de Zaragoza

Madrid, Noviembre 2023

Universidad Zaragoza 1542

Departamento de Informática e Ingeniería de Sistemas Universidad Zaragoza

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Proyecto Investigador: Investigación en Curso y Líneas Futuras

- **Equipos heterogéneos:** Dynamic Region Coverage
- Prealimentación y feedback para mejorar la velocidad de seguimiento de la region

A Distributed Robot Swarm Control
for Dynamic Region Coverage

PROBLEM: Cover a (fastly) moving region
evenly and preventing collisions (Voronoi-based)
with a large group of n autonomous robots
that communicate only with a few neighbors
(within a radius r)

A Practical Method to Cover Evenly
a Dynamic Region with a Swarm

Enrique Teruel
Rosario Aragues
Gonzalo López-Nicolás

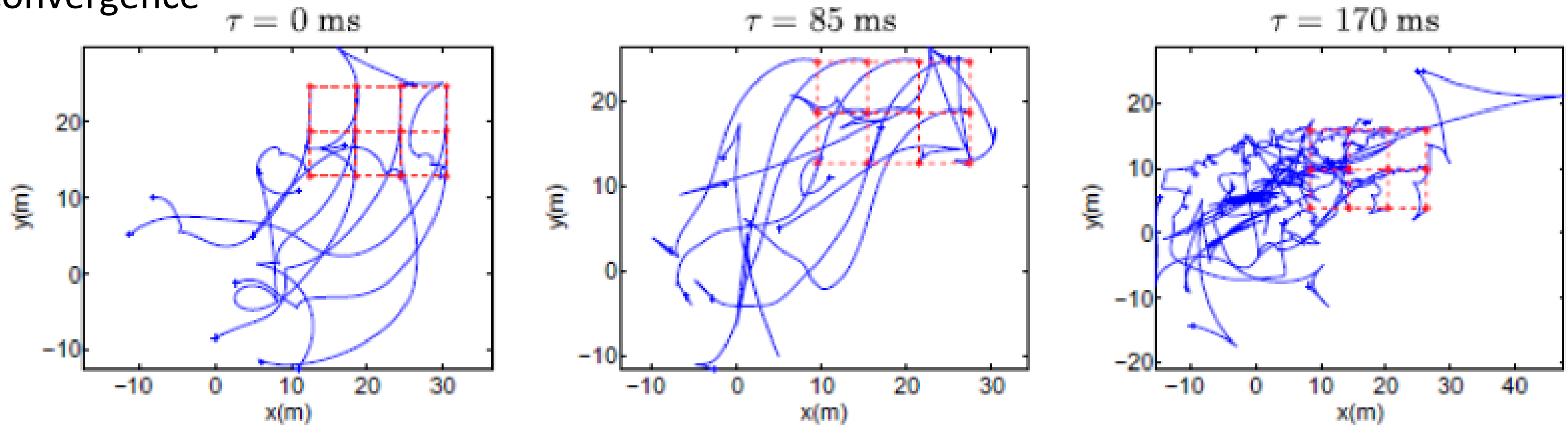
Universidad de Zaragoza

[E. Teruel, R. Aragues, G. López-Nicolás. *A distributed robot swarm control for dynamic region coverage*. Robotics and Autonomous Systems, 119:51–63, 2019]

[E. Teruel, R. Aragues, G. López-Nicolás. *A Practical Method to Cover Evenly a Dynamic Region with a Swarm*. IEEE Robotics and Automation Letters 2021, Accepted]

Proyecto Investigador: Investigación en Curso y Líneas Futuras

Publicaciones recientes en estas líneas:Effects of **communication delays** in the convergence



[A. González, R. Aragues, G. López-Nicolás, C. Sagues. *Stability analysis of nonholonomic multiagent coordinate-free formation control subject to communication delays*. International Journal of Robust and Nonlinear Control, 28(14):4121–4138, 2018]

[A. González, R. Aragues, G López-Nicolás, C. Sagues. *Formation control synthesis in local frames under communication delays and switching topology: An LMI approach*. American Control Conference (ACC), pp. 5328–5333, 2019]

[A. González, R. Aragues, G. López-Nicolás, C. Sagues. *Predictor–feedback synthesis in coordinate–free formation control under time–varying delays*. Automatica,113:108811, 2020]

[A. González, R. Aragues, G. López-Nicolás, C. Sagues. *Weighted predictor feedback formation control in local frames under time-varying delays and switching topology*. International Journal of Robust and Nonlinear Control,]30(8):3484–3500, 2020}