Robust MPC design, Future and Practical Applications

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Outline

Woody Allen:

"I took a speed reading course and read 'War and Peace' in twenty minutes. …

….. It involves Russia."

MPC successful in industry.

- Many and very diverse and successful applications:
  - Petrochemical, polymers,
  - Semiconductor production,
  - Air traffic control
  - Clinical anesthesia,
  - ….
  - Life Extending of Boiler-Turbine Systems via Model Predictive Methods, Li et al (2004)
- Many MPC vendors

MPC successful in Academia

- Many MPC sessions in control conferences (2/12 at this symposium) and control journals, MPC workshops.
- MPC in other research areas: industrial electronics, chemical engineering, energy, transport …
- 4/8 finalist papers for the IFAC journal *CEP best paper award* were MPC papers (2/3 finally awarded were MPC papers)

IFAC Pilot Industry Committee

Chaired by Tariq Samad (Honeywell), 28 total: 15 industry, 12 academia, 1 gov't;

Members asked to assess impact of several advanced control technologies:

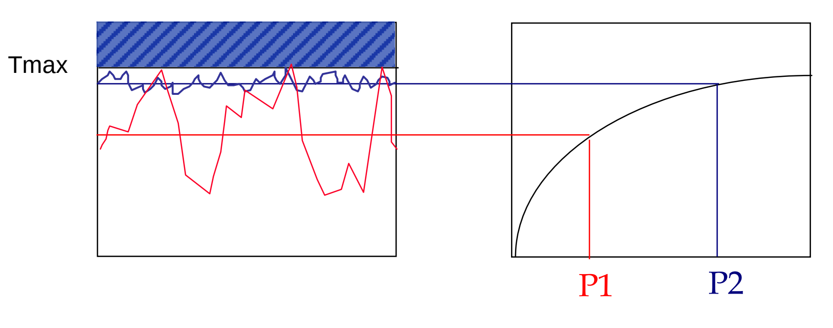
- **Q1 Responses [23 responses]**
  - **PID control: 23 High-impact**
  - **Model-predictive control: 18 High-impact; 2 No/Lo impact**
  - **System identification: 14 High-impact; 2 No/Lo impact**
  - **Process data analytics: 14 High-impact; 4 No/Lo impact**
  - **Soft sensing: 12 High-impact; 5 No/Lo impact**
  - **Fault detection and identification [22]: 11 High-impact; 4 No/Lo impact**
  - **Decentralized and/or coordinated control: 11 High-impact; 7 No/Lo impact**
  - **Intelligent control: 8 High-impact; 7 No/Lo impact**
  - **Discrete-event systems [22]: 5 High-impact; 7 No/Lo impact**
  - **Nonlinear control: 5 High-impact; 8 No/Lo impact**
  - **Adaptive control: 4 High-impact; 10 No/Lo impact**
  - **Hybrid dynamical systems: 3 High-impact; 10 No/Lo impact**
  - **Robust control: 3 High-impact; 10 No/Lo impact**

Why is MPC so successful ?

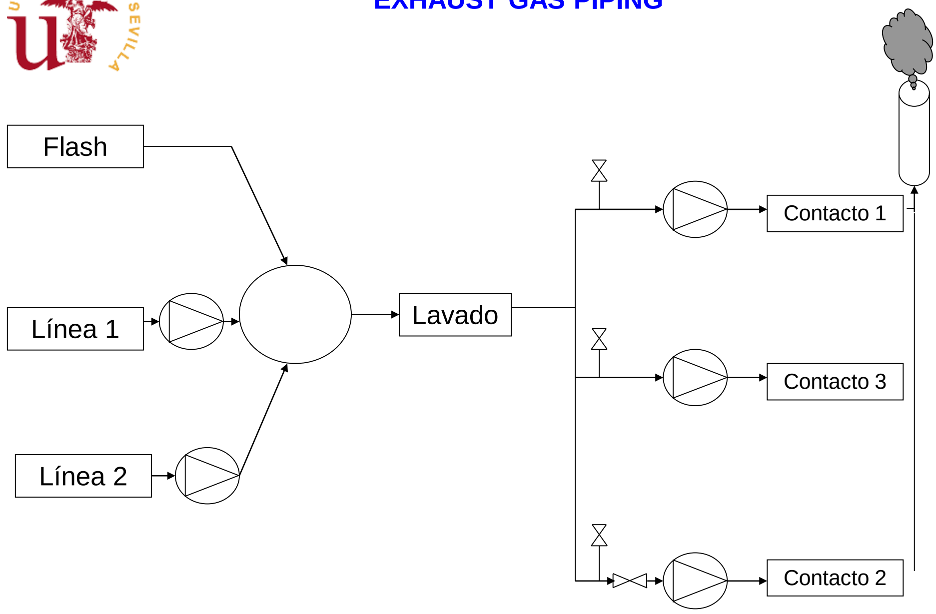
- MPC is Most general way of posing the control problem in the time domain:
  - Optimal control
  - Stochastic control
  - Known references
  - Measurable disturbances
  - Multivariable
  - Dead time
  - Constraints
  - Uncertainties

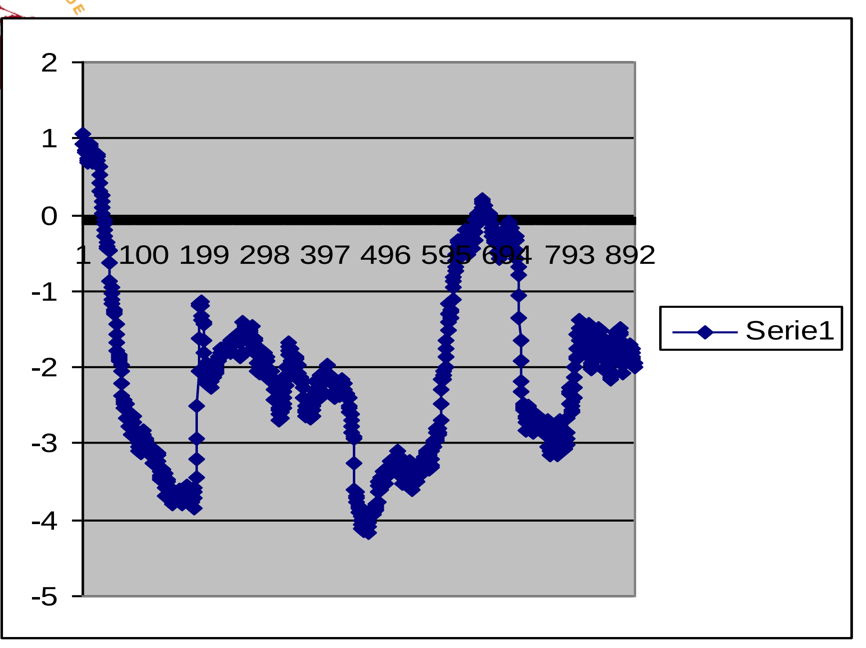
Real reason of success: Economics

- MPC can be used to optimize operating points (economic objectives). Optimum usually at the intersection of a set of constraints.
- Obtaining smaller variance and taking constraints into account allow to operate closer to constraints (and optimum).
- Repsol reported 2-6 months payback periods for new MPC applications.



EXHAUST GAS PIPING





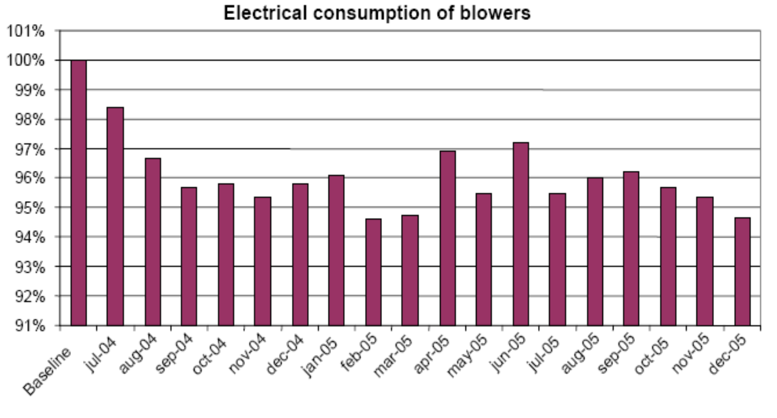


Fig. 14. Electrical consumption reduction.

Benefits

- Yearly saving of more that 1900 MWh
- Standard deviation of the mixing chamber pressure reduced from 0.94 to 0.66
- Operator's supervisory effort: percentage of time operating in auto mode raised from 27% to 84%.

A little bit of history: the beginning

- Kalman, LQG (1960)
- Propoi, "Use of LP methods ..." (1963)
- Richalet *et al*, Model Predictive Heuristic Control (MPHC) IDCOM (1976, 1978) (150.000 $/year benefits because of increased flowrate in the fractionator application)
- Cutler & Ramaker, DMC (1979,1980)
- Cutler *et al* QDMC (QP+DMC) (1983)
- Clarke *et al* GPC (1987)
- First book: Bitmead *et al*, (1990)

The impulse of the 90s. A renewed interest from Academia (stability)

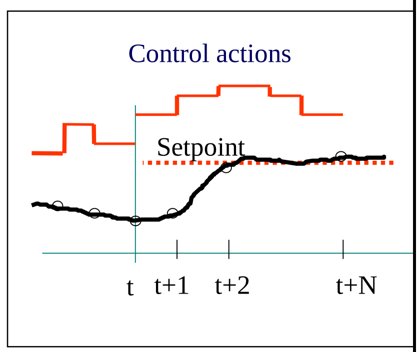
- Stability was difficult to prove because of the ***finite horizon and the presence of constraints*** (non linear controller, no explicit solution, …)
- A **breakthrough** produced in the field. As pointed out by Morari: "***the recent work has removed this technical and to some extent psychological barrier (people did not even try) and started wide spread efforts to tackle extensions of this basic problem with the new tools*"**. (Rawlings & Muske, 1993)
- Many contributions to stability and robustness of MPC: Allgower, Campo, Chen, Jaddbabaie, Kothare, Limon, Magni, Mayne, Michalska, Morari, Mosca, de Nicolao, de Olivera, Scattolini, Scokaert…

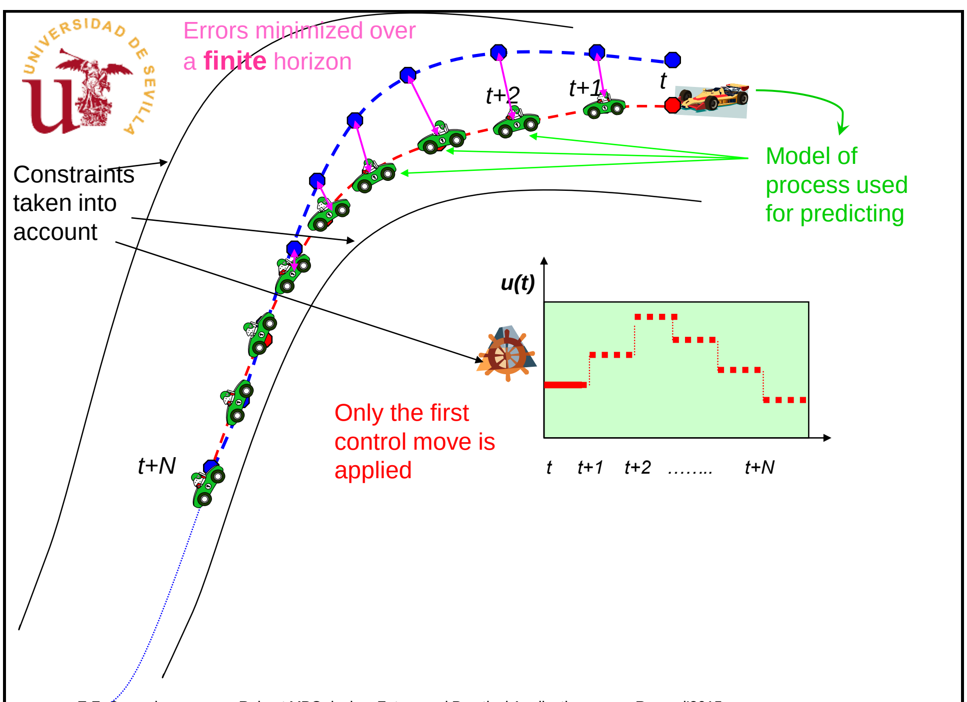
MPC now

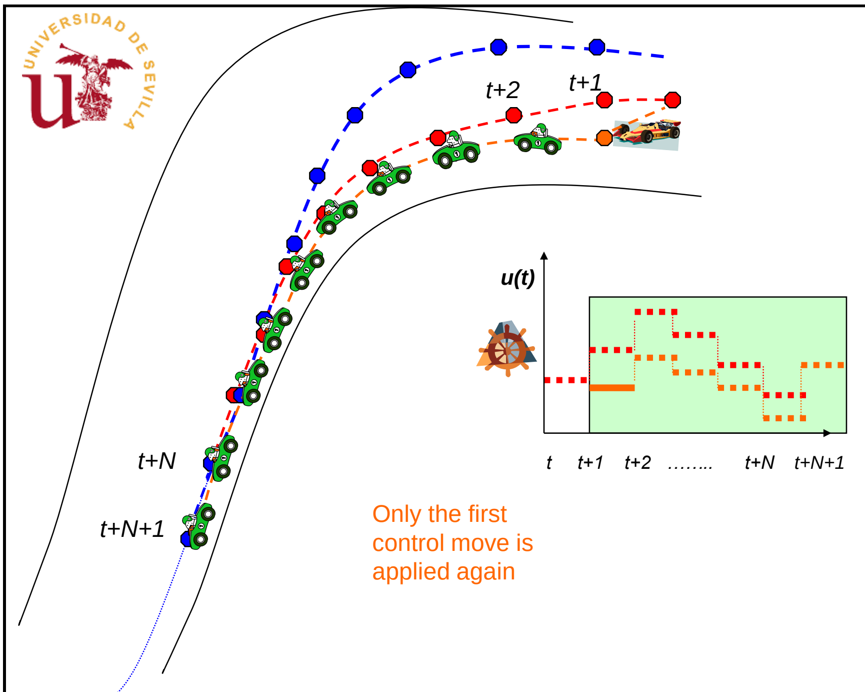
- Linear MPC is a mature discipline. More than 30.000 industrial applications.
- The number of applications seems to duplicate every 4 years.
- Some vendors have NMPC products: Adersa (PFC), Aspen Tech (Aspen Target), Continental Control (MVC), DOT Products (NOVA-NLC), Pavilon Tech. (Process Perfecter)
- Efforts to develope MPC for more difficult situations:
  - Multiple and logical objectives (Morari, Floudas)
  - Hybrid processes (Morari, Bemporad, Borrelli, De Schutter, van den Boom …)
  - Nonlinear (Alamir, Alamo, Allgower, Biegler, Bock, Bravo, Chen, De Nicolao, Findeisen, Jadbadbadie, Limon, Magni, …)
  - Fast MPC (Bemporad, Löfberg, Fikar, ...)
- Challenge: Incorporate stability and robustness issues in industrial MPC design.

MPC strategy

- At sampling time *t the future control sequence is compute so that the future sequence of predicted output y(t+k/t) along a horizon N follows the future references as best as possible.*
- The first control signal is used and the rest disregarded.
- The process is repeated at the next sampling instant *t+1*

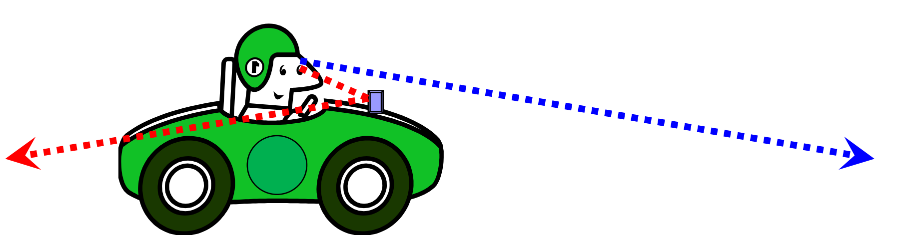






Only the first
control move is
applied again

MPC vs. PID



PID: $u(t)=u(t-1)+g_0\,e(t) + g_1\,e(t-1) + g_2\,e(t-2)$

MPC strategy

Consider a nonlinear invariant discrete time system:

$$x^+=f(x,u), \quad x \in R^n, u \in R^m$$

The system is subject to hard constraints

$$x \in X, u \in U$$

Let $\mathbf{u}=\{u(0),\ldots,u(N-1)\}$ be a sequence of N control inputs applied at $x(0)=x$,

the predicted state at i is

$$x(i)=\Phi(i;x,\mathbf{u})=f(x(i-1), u(i-1))$$

MPC strategy

1. Optimization problem $P_N(x,\Omega)$:

$$u^*= \arg\min_u \sum_{(i=0,\ldots,N-1)} l(x(i),u(i)) + F(x(N))$$

- Operating constaints .
  $x(i) \in X, u(i) \in U, \; i=0,\ldots,N-1$
- Terminal constraint (stability): $x(N) \in \Omega$

2. Apply the receding horizon control law:
   $K_N(x)=u^*(0)$.

Linear MPC

- ***f(x,u)*** is an affine function (model)
- ***X,U,Ω*** are polyhedra (constraints)
- ***l*** and ***F*** are quadratic functions ***(***or ***1-norm*** or ***∞-norm*** functions***)***

$$\Downarrow$$

- **QP or LP**

Otherwise

- If ***f(x,u)*** is not an affine function
- Or any of ***X,U,Ω*** are not polyhedra
- Or any of ***l*** and ***F*** are not quadratic functions ***(***or ***1-norm*** or ***∞-norm*** functions***)***

$$\Downarrow$$

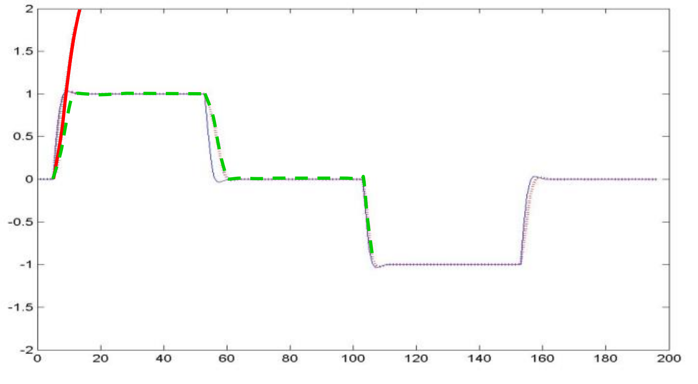
- Non linear MPC (NMPC)
- Non linear (non necessarily convex) optimization problem much more difficult to solve.

MPC stability and constraints

- Stability was difficult to prove because of the ***finite horizon and the presence of constraints*** (non linear controller, no explicit solution, …)
- Manipulated variables can always be kept in bound by the controller by clipping the control action or by the actuator.
- Output constraints are mainly due to safety reasons, and ***must be controlled in advance*** because output variables are affected by process dynamics.
- Not considering contraints properly may lead to unstability
- Gunter Stein: "Respect the unstable"

Stability and constraints

$$y(t+1)=1.2\,y(t)+0.2\,u(t-2) \quad \text{with } -4 < u(t) < 4, \; N=5$$



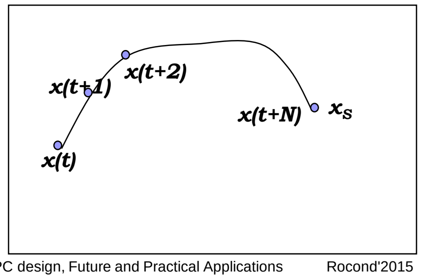
MPC stability

- Infinite horizon. Keerthi and Gilbert (J. Optim.Theory Appl., 1988) the objective function can be considered a Lyapunov function, providing nominal stability. Cannot be implemented: an infinite set of decision variables.

- Terminal state equality constraint. Clarke and Scattolini (IEE, 1991)

  $x(k+N)= x_S$

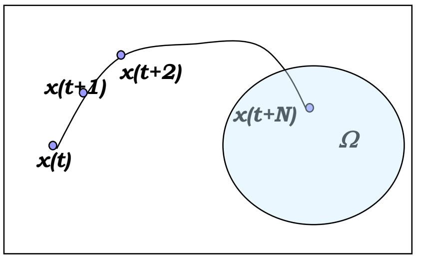
  difficult to implement in practice.



MPC stability (2)

- Dual control. Michalska and Mayne (1993) $x(N) \in \Omega$

  Once the state enters Ω the controller switches to a previously computed stable linear strategy.



- Quasi-infinite horizon. Chen and Allgower (1998). Terminal region and stabilizing control, but only for the computation of the terminal cost. The control action is determined by solving a finite horizon problem without switching to the linear controller even inside the terminal region. The term $(\| x(t+N)\|_P)2$ added to the cost function and approximates the infinite- horizon one.

MPC stability (3)

- **Asymptotic stability theorem** (Mayne 2001)
- The terminal set Ω is a control invariant set.
- The terminal cost ***F(x)*** is an associated Control Lyapunov function such that

  $min_{\{\mathbf{u} \in \mathbf{U}\}} \{F(f(x,u))-F(x) + l(x,u) \mid f(x,u) \in \Omega\} \leq 0 \ \forall x \in \Omega$

- Then the closed loop system is asymptotically stable in $X_N(\Omega)$

How robust is the *stable* MPC ?

stable MPC is Input to State Stable

- Theorem:
  - Assume that

MPC is inherently robust under mild conditions: continuity of *f(x,u,d,w)*

- Corollary: Local ISS

  Uniform continuity can be relaxed to continuity at $x = 0$, $u = 0$, $d = 0$ and $w = 0$.

D. Limón, T. Alamo and E.F. Camacho, Input to State Stable MPC for Constrained Discrete-time Nonlinear Systems with Bounded Additive Uncertainties, 2012, Las Vegas.

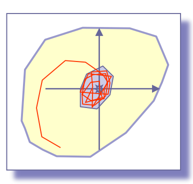
D. Limon. T. Alamo, D.M. Raimondo, D. Muñoz de la. Peña. J.M. Bravo and E.F. Camacho, Input-to-state stability: a unifying framework for robust model predictive control, Nonlinear Model Predictive Control Lecture Notes in Control and Information Sciences Volume 384, 2009, pp 1-26

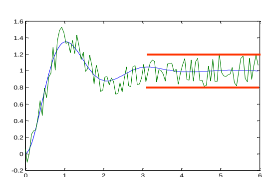
Uncertainties in MPC

- Past and present:
  - Model
  - State
- Future
  - Model
  - Process load
  - References and Control objectives

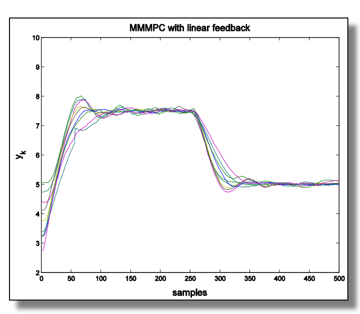
Robustness in MPC

- Robust stability.





- Robust constraint satisfaction.
- Robust performance.
- Robustness to failures.



Robustness

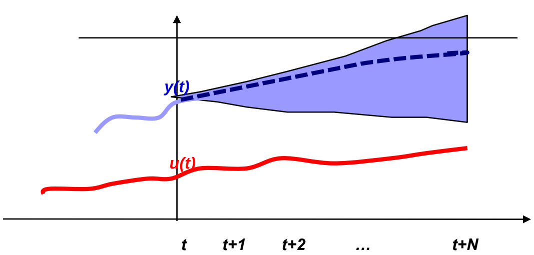
Uncertain system: $x^+ = f(x,u,\theta)$, $x \in R^n$, $u \in R^m$ $\theta \in R^p$

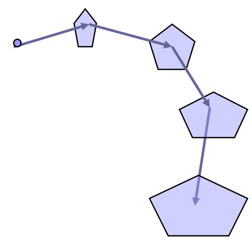
With bounded uncertainties $\theta \in \Theta$ and subject to hard constraints $x \in X$, $u \in U$

The **uncertain evolution sets or reachable sets (tube)**:

$$X(i) = \Gamma(i;x,u) = \{z \in R^n \mid \exists\, \theta \in \Theta,\ y \in X(i-1),\ z = f(y, u(i-1), \theta)\}$$

and $X(0)=x$

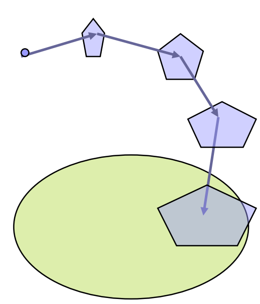




Robust stability

- The stability conditions has to be satisfied for all possible values of the uncertainties.
- The terminal set Ω is a **robust** control invariant set. (i.e. $\forall x \in \Omega,\ \forall \theta \in \Theta$ $\exists\, u \in U \mid f(x,u,\theta) \in \Omega$)
- The terminal cost $F(x)$ is an associated Control Lyapunov function such that

  $\min_{\{u \in U\}} \{F(f(x,u,\theta)) - F(x) + l(x,u) \mid f(x,u,\theta) \in \Omega\} \le 0\ \forall x \in \Omega,\ \forall \theta \in \Theta$



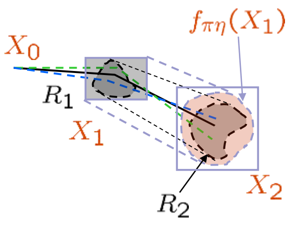
Computation of reachable sets and invariant sets for robust constraint satisfaction or robust stability

- A sequence of sets $\{X_0, X_1, \cdots, X_N\}$ is a

  Sequence of reachable sets (or tube)

  for a given sequence of control inputs $\mathbf{v}$, if

  $$f_{\pi\eta}(X_i, v(i), D, W_\eta) \subseteq X_{i+1}$$

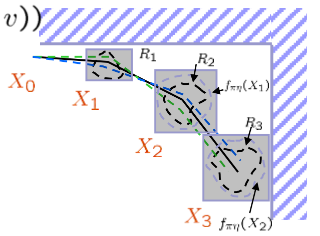


where $f_{\pi\eta}(x, v, d, w_\eta) \triangleq f_\pi(x, v, d, w_\eta \eta_\pi(x, v))$

  - If $R_0 \subseteq X_0$, then $R_j \subseteq X_j$

- Robust feasibility is ensured if

  $$\boxed{X_j \times v(j) \subseteq Z_\pi}$$



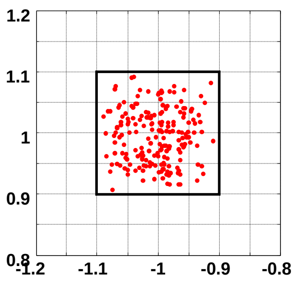
Computation of reachable sets and invariant sets for robust constraint satisfaction or robust stability

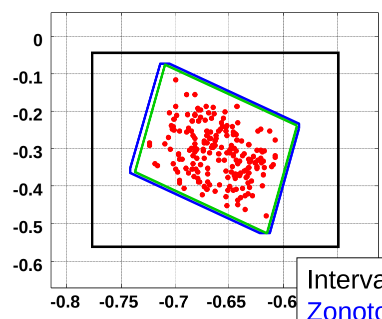
- Reachable sets are difficult to compute.
- Approximations and bounding based on:
  - Ellipsoids
  - Linealization
  - Lipschitz continuity
  - Interval Arithmetic
  - Zonotopes
  - DC Programming

Illustrative example

- Illustrative example (Alamo'08):

$$\begin{aligned} x_1^+ &= -0.7x_2 + 0.1x_2^2 + 0.1x_1x_2 + 0.1e^{x_1} \\ x_2^+ &= x_1 + x_2 - 0.1x_1^2 + 0.2x_1x_2 \end{aligned}$$

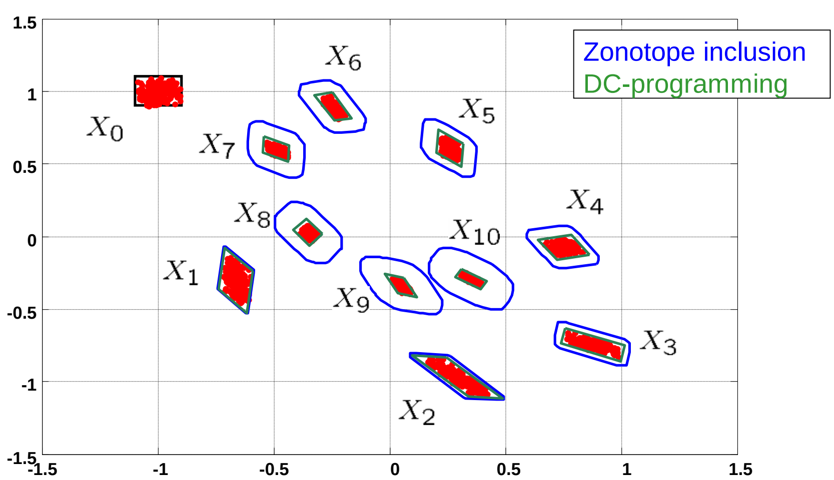




One step set

Illustrative example

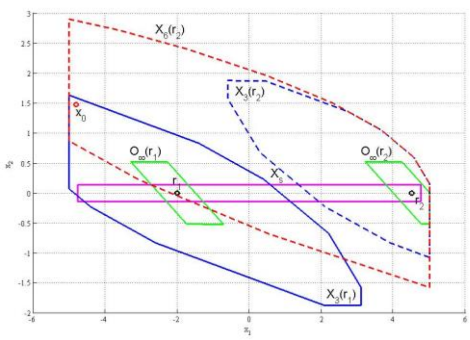
- Sequence of reachable sets



MPC for tracking (motivation)

- Most stability – robustness results for the origin.
- What if your setpoints change ?

Moving the invariant set to the new setpoint may not work in the presence of constraints



Robust MPC for Tracking

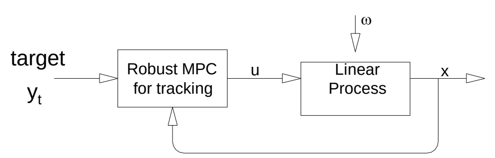
Problem description

Consider the following discrete time LTI system with additive bounded uncertainties:

$$x^+ = Ax + Bu + w$$

The system is constrained to:

$$x \in \mathcal{X} \subset \mathbb{R}^n$$
$$u \in \mathcal{U} \subset \mathbb{R}^m$$
$$w \in \mathcal{W} \subset \mathbb{R}^n$$



Objective: Given any admissible setpoint ***s***, design a control law such that:

- ➢y(k) tends to the neighbourhood of $\boldsymbol{y_t}$ when k→∞
- ➢x(k) and u(k) are admissible for all k ≥ 0 and all possible realizations of ω

Robust MPC for Tracking

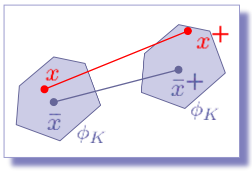
Lemma *(Langson 2004)*

$$\left.\begin{array}{llcl} \text{Nominal model:} & \bar{x}^+ & = & A\bar{x} + B\bar{u} \\ \text{Plant model:} & x^+ & = & Ax + Bu + w \\ \text{Control law:} & u & = & K(x-\bar{x}) + \bar{u} \\ \text{Control error:} & e & = & x - \bar{x} \end{array}\right\} e^+ = \underbrace{(A+BK)}_{Hurwitz} e + w$$

Robust Positively invariant (RPI) set ϕ_K

Consider that $(A+BK)$ is Hurwitz.
If $e \in \phi_K$, then $e^+ \in \phi_K$ for all $w \in \mathcal{W}$

$$x \in \bar{x} \oplus \phi_K \Rightarrow x^+ \in \bar{x}^+ \oplus \phi_K$$

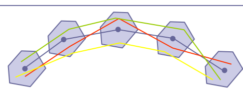


Robust MPC for Tracking

The tube: *(Langson 2004 ; Bertsekas 1972)*

Recursively:

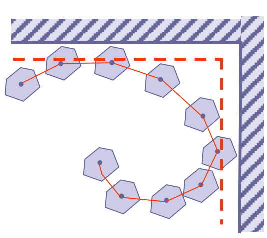
If $x(0) \in \bar{x}(0) \oplus \phi_K$, then $x(i) \in \bar{x}(i) \oplus \phi_K$ $\forall i \geq 0$



(Mayne et al., 2005)

- Considering the tighter set of constraints for the nominal system

$$\begin{array}{lcl} \bar{\mathcal{X}} & = & \mathcal{X} \ominus \phi_K \\ \bar{\mathcal{U}} & = & \mathcal{U} \ominus K\phi_K \end{array}$$



Applying $u(i) = K(x(i) - \bar{x}(i)) + \bar{u}(i)$

$$\left.\begin{array}{lcl} \bar{x}(0) & \in & x(0) \oplus (-\phi_K) \\ \bar{u}(i) & \in & \bar{U}, \quad i \geq 0 \\ \bar{x}(i) & \in & \bar{X}, \quad i \geq 0 \end{array}\right\} \begin{array}{lcl} x(i) & \in & X \\ u(i) & \in & U \end{array}$$

MPC vs Robust MPC for tracking

$$\min_{\mathbf{u},\bar{\theta}} \sum_{i=0}^{N-1} (\|x(i)-\bar{x}_s\|_Q^2 + \|u(i)-\bar{u}_s\|_R^2) + \|x(N)-\bar{x}_s\|_P^2 + \|\bar{y}_s - y_t\|_T^2$$

$$s.t. \quad u(j) \in \mathcal{U},\ x(j) \in \mathcal{X}, \qquad j = 0,\cdots,N-1.$$

$$(x(N),\bar{\theta}) \in \Omega_t^a.$$

$$(\bar{x}_s,\bar{u}_s) = M_\theta \bar{\theta}$$
$$\bar{y}_s = C\bar{x}_s + D\bar{u}_s$$

$$\min_{\mathbf{u},\bar{\theta},\bar{x}} \sum_{i=0}^{N-1} (\|\bar{x}(i)-\bar{x}_s\|_Q^2 + \|\bar{u}(i)-\bar{u}_s\|_R^2) + \|\bar{x}(N)-\bar{x}_s\|_P^2 + \|\bar{y}_s - y_t\|_T^2$$

$$s.t. \quad \bar{x} \in x \oplus (-\phi_K)$$

$$\bar{u}(j) \in \bar{\mathcal{U}},\ \bar{x}(j) \in \bar{\mathcal{X}}$$

$$(\bar{x}(N),\bar{\theta}) \in \Omega_t^a. \qquad j = 0,\cdots,N-1.$$

$$(\bar{x}_s,\bar{u}_s) = M_\theta \bar{\theta}$$
$$\bar{y}_s = C\bar{x}_s + D\bar{u}_s$$

Robust MPC for Tracking

Theorem: Consider that

- $\bar{K}$ is such that $(A+B\bar{K})$ is stable
- Q>0, R>0, $\bar{K}$ and P such that: $P-(A+B\bar{K})^T P(A+B\bar{K}) = Q+\bar{K}^T R\bar{K}$
- Ω^a_t is an admissible invariant set for tracking for the nominal system subject to the following constraints $\bar{x}(i) \in \bar{\mathcal{X}}$ and $\bar{u}(i) \in \bar{\mathcal{U}}$
- $T > 0$
- **K** is such that (A+B**K**) is stable and $\bar{\mathcal{X}}$, $\bar{\mathcal{U}}$ are not empty sets

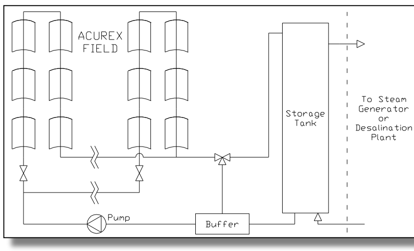
Let $\mathcal{X}_N = \bar{\mathcal{X}}_N \oplus \phi_K$ be the feasibility region of the optimization problem

Then, for any feasible initial state i.e., $x \in \Xi_N$ and any reachable target, the uncertain system is steered asymptotically to the set $y_t \oplus (C+DK)\phi_K$ for all possible realization of the disturbances, satisfying the constraints

(Alvarado, Limon, Camacho, 2010)

PSA solar plant

Located in Taberna desert (Almeria, Spain).
Hot oil that can be used to produce steam to produce electricity or for a desalination plant

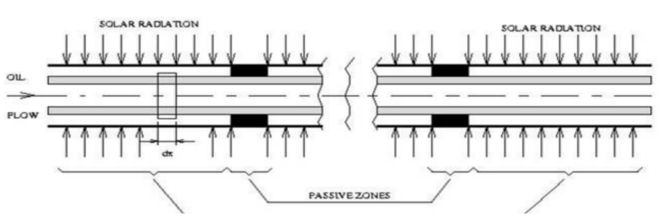


The control goal is to keep the oil's temperature close to the reference.

Process model

Metal: $\rho_m C_m A_m \partial T_m / \partial t = \eta_o I G - G H_l (T_m - T_a) - L H_t (T_m - T_f)$

Fluid: $\rho_f C_f A_f \partial T_f / \partial t + \rho_f C_f q \partial T_m / \partial x = L H_t (T_m - T_f)$

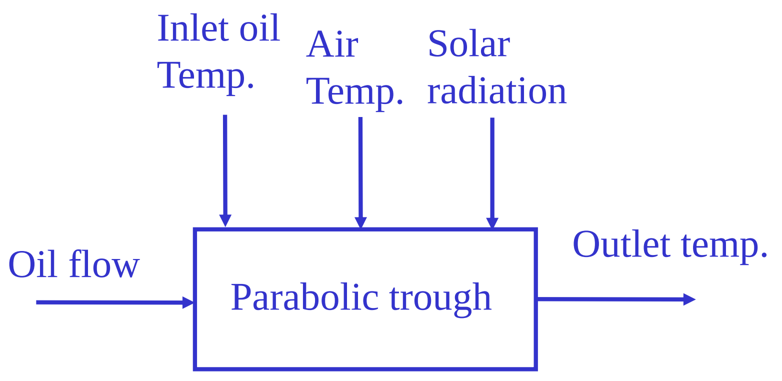


Simulink model can be downloaded from:

E.F. Camacho, *et al.* Control of Solar Energy Systems, Springer, 2014

http://www.esi2.us.es/~eduardo/libro-s/libro.html

Solar field



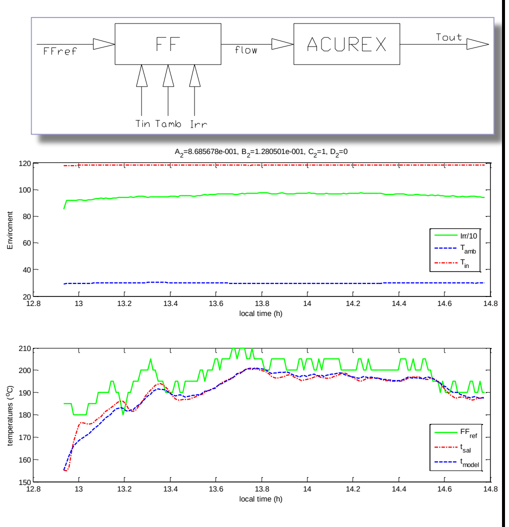
PSA trough solar plant

Identification:

$$y = x = T_{out}$$
$$u = FF_{ref}$$

The first order model

$$x^{+} = 0.8656x + 0.1251u + w$$



PSA trough solar plant

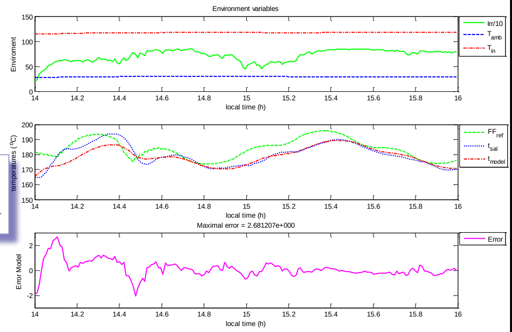
Identification:

To determine the set W the output of the model is compared with the real output for a big set of data.

$$\mathcal{W} = \{w \in \mathbb{R}^1 : \|w\|_\infty \leq 5\}$$

The constraints sets are:

$$\mathcal{X} = \{x \in \mathbb{R}^1 : 0 \leq x \leq 300\}$$
$$\mathcal{U} = \{u \in \mathbb{R}^1 : 100 \leq u \leq 350\}$$

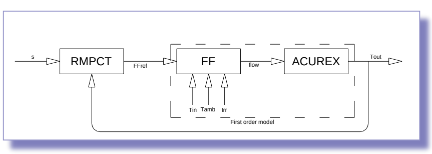


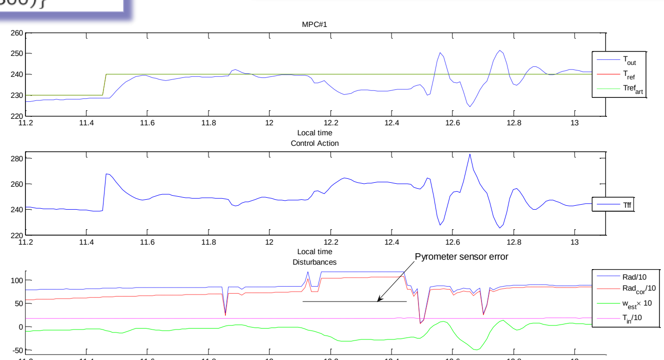
Plant controlled by robust MPC for tracking

Controller parameters:

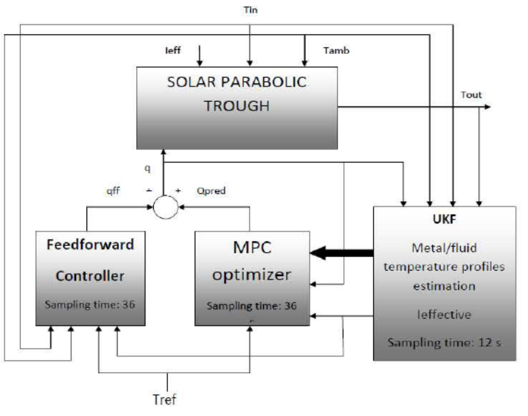
MPC#2

$Q_\phi = 10,\ R_\phi = 1,\ K = -1.81804$
$\phi_K = \{e \in \mathbb{R}^n : |e| < 13.7275\}$
$Q = 1000,\ R = 1,\ \bar{K} = -6.40848$
$\mathcal{S} = \{s \in \mathbb{R}^p : (121.62 \leq s \leq 258.89)\}$
$\bar{\mathcal{U}} = \{\bar{u} \in \mathbb{R}^m : (124.96 \leq \bar{u} \leq 325.04)\}$
$\mathcal{X}_n = \{x \in \mathbb{R}^n : (0 \leq x \leq 300)\}$



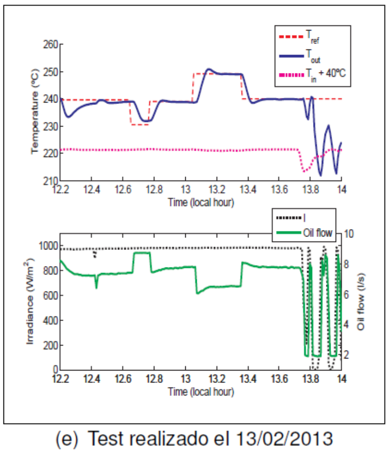


UKF MPC

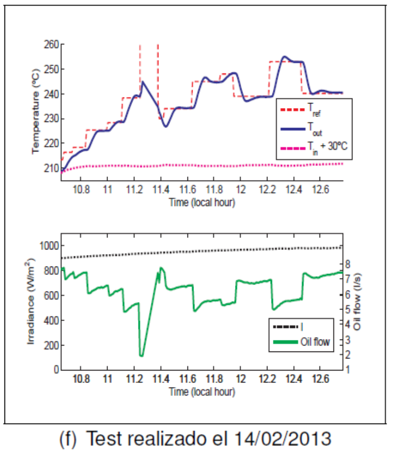


A.J.Gallego, E.F. Camacho (2012,2013)

UKF NMPC

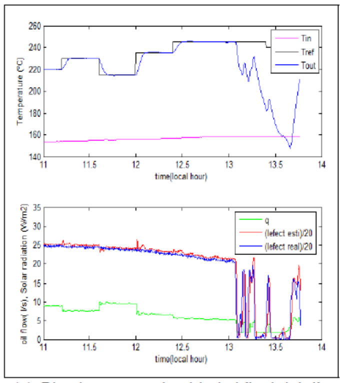


(e) Test realizado el 13/02/2013

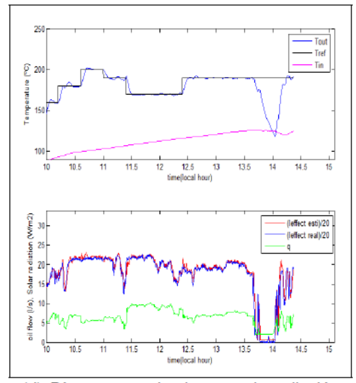


(f) Test realizado el 14/02/2013

PSA Acurex



(c) Dia claro con nubosidad al final del día



(d) Día con perturbaciones en la radiación

Abengoa trough plants

Spain (650 MWe)
- Solucar (3x50MWe)
- Helioenergy (2x50MWe)
- Solacor (2x50MWe)
- Helios (2x50MWe)
- Solaben (4x50MWe)

USA (560 MWe)
- Solana (280 MWe)
- Mojave (2x140 MWe)

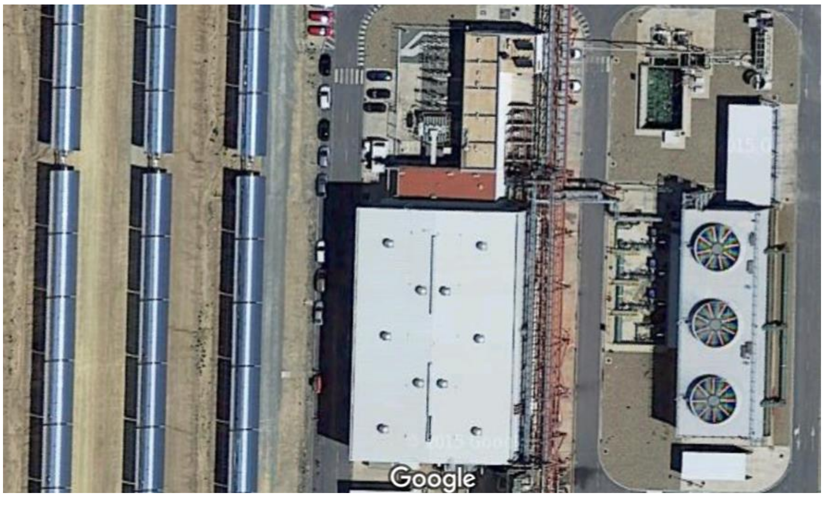
South Africa (100 MWe)
- Kaxu (100 MWe)

Arabs Emirates (100 MWe)
- Sham1 (100 MWe)

Argelia (20 MW)
- Hassi R'Mel (co-generation)

Solucar (3x50MWe)

Solucar (3x50MWe)



Outline

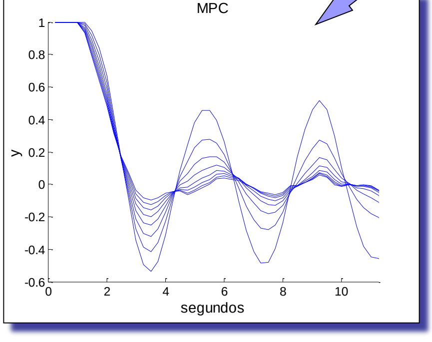
1. Model Predictive Control
2. Stability and robustness for MPC
3. Min max MPC
4. Fault tolerant MPC
5. Conclusions

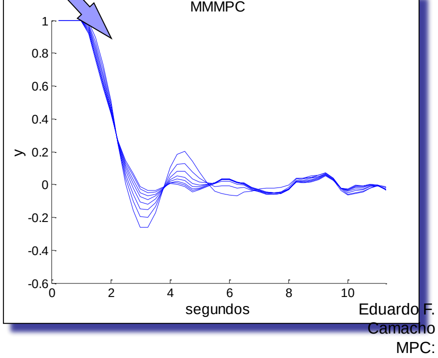
Why Min-Max Model Predictive Control ?

$$G(s) = \frac{1}{(s+1)(s+p)}$$

$$p \in [-0.4, 1]$$

Better performance against uncertainties → More Robustness





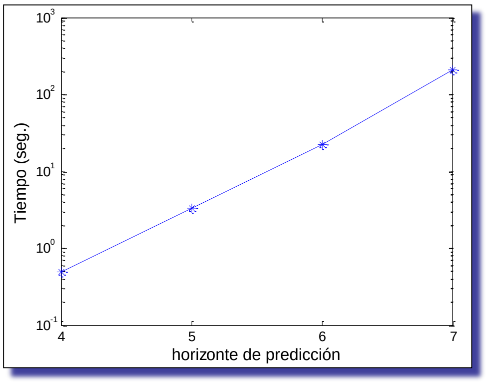
Why Min-Max Model Predictive Control ?

In spite of advantages, the number of reported applications is very low

Berenguel *et al.* 1997
Kim *et al. 1998*
Álvarez *et al. 2003*

¿?

Computational burden



Open loop vs close loop prediction

MPC with open-loop prediction: The sequence of control actions is computed with the information available at time t.

- 1987 (Campo and Morari).
- Min-max over real numbers
- Conservatism.
- Techniques available.

MPC with close-loop prediction: The controller considers that the value of the disturbances will be known in the future.

- 1997 (Lee and Yu) and 1998 (Scokaert et al)
- Min-max over control laws.
- Less conservative
- Greater computational burden (not any single reported application to a real process).

Robust Model Predictive Control

System model:

$$x(t+1) = Ax(t) + Bu(t) + D\theta(t+1)$$
$$y(t) = Cx(t)$$

$$\|\theta(t)\|_\infty \leq \epsilon$$

Bounded additive uncertainties

Two strategies to consider u(t):

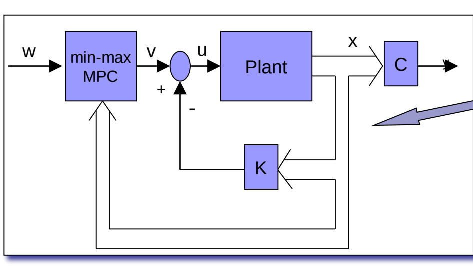
- Open-loop predictions: $u(t), u(t+1), u(t+2), \ldots$
- Semi-feedback predictions:

$$u(t) = -Kx(t) + v(t)$$

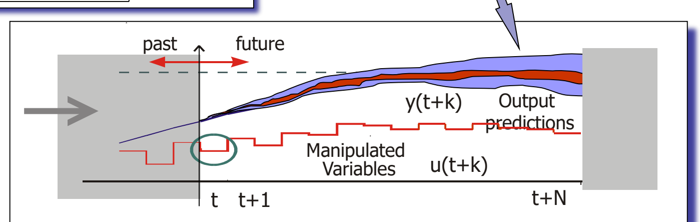
$$x(t+1) = A_{CL}x(t) + Bv(t) + D\theta(t+1),$$
$$A_{CL} = A - BK$$

Computed by the controller

Robust MPC



The inner loop pre-stabilizes the nominal system



Min-max MPC open loop (1-norm)

$$J(\mathbf{u},\theta)=\sum_{j=N_1}^{N_2}\sum_{i=1}^{n}|y_i(t+j\mid t,\theta)-w_i(t+j)|+\lambda\sum_{j=1}^{N_u}\sum_{i=1}^{m}|\triangle u_i(t+j-1)| \quad (1)$$

If a series of $\mu_i \geq 0$ and $\beta_i \geq 0$ such that for all $\theta \in \Theta$,

$$-\mu_i \leq (y_i(t+j)-w_i(t+j)) \leq \mu_i$$
$$-\beta_i \leq \triangle u_i(t+j-1) \leq \beta_i$$
$$0 \leq \sum_{i=1}^{n\times N}\mu_i + \lambda\sum_{i=1}^{m\times N_u}\beta_i \leq \gamma$$

then γ is an upper bound of

$$\mu^*(\mathbf{u})=\max_{\theta\in\mathcal{E}}\sum_{j=1}^{n}\sum_{i=1}|y_i(t+j,\theta)-w_i(t+j)|+\lambda\sum_{j=1}^{N_u}\sum_{i=1}^{m}|\triangle u_i(t+j-1)|$$

Min-max MPC open loop (1-norm)

$$\min_{\gamma,\mu,\beta,\mathbf{u}} \gamma$$

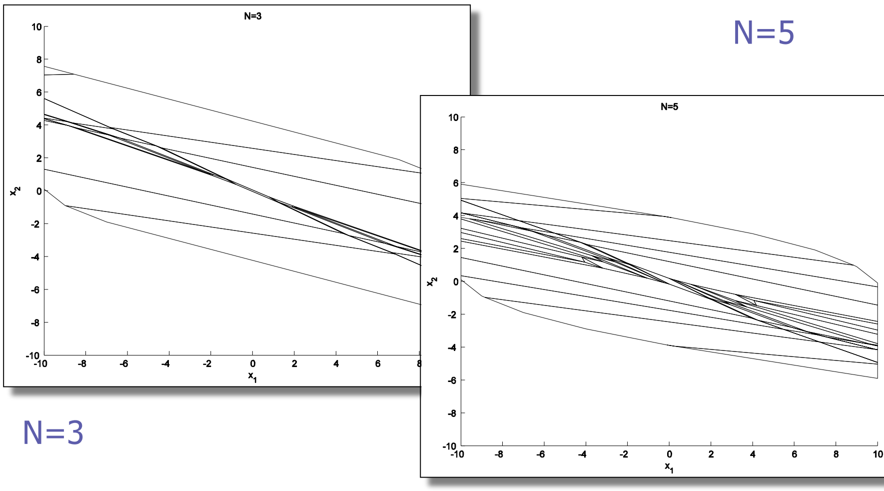
subject to

$$\left.\begin{array}{rcl}\mu & \geq & G_u\mathbf{u}+G_\theta\theta+\mathbf{f}+\mathbf{w}\\ \mu & \geq & -G_u\mathbf{u}-G_\theta\theta-\mathbf{f}+\mathbf{w}\\ \overline{\mathbf{y}} & \geq & G_u\mathbf{u}+G_\theta\theta+\mathbf{f}\\ -\underline{\mathbf{y}} & \geq & -G_u\mathbf{u}-G_\theta\theta-\mathbf{f}\end{array}\right\}\ \forall\theta\in\mathcal{E}$$

$$\begin{array}{rcl}\beta & \geq & \mathbf{u}\\ \beta & \geq & -\mathbf{u}\\ \overline{\mathbf{u}} & \geq & \mathbf{u}\\ -\underline{\mathbf{u}} & \geq & \mathbf{u}\\ \overline{\mathbf{U}} & \geq T & \mathbf{u}+\mathbf{1}u(t-1)\\ -\underline{\mathbf{U}} & \geq & -T\mathbf{u}-\mathbf{1}u(t-1)\\ \gamma & \geq & \mathbf{1}^t\mu+\lambda\mathbf{1}\beta\end{array}$$

LP (with many constraints: the vertices of the uncertainty polytope)

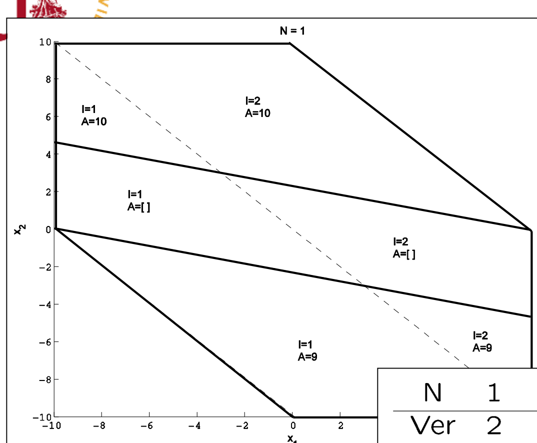
Multiparametric min-max MPC



N=5

N=3

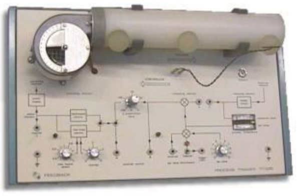
Reduction of computational burden



Set of active vertices is very small

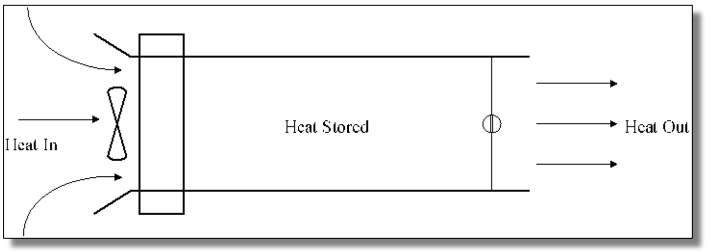
N	1	3	5	6	8	10
Ver	2	8	32	64	256	1024
Act	6	8	10	12	14	16
Reg	4	45	71	97	147	201

Feedback PT-326



- Scaled laboratory process
- 2nd order system
- Fast dynamics
- Ts = 0.4 s.

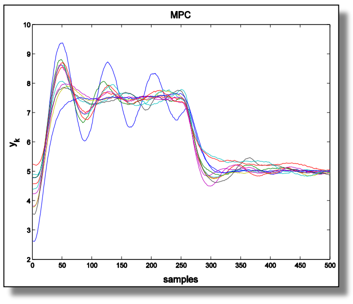
(Explicit solution)



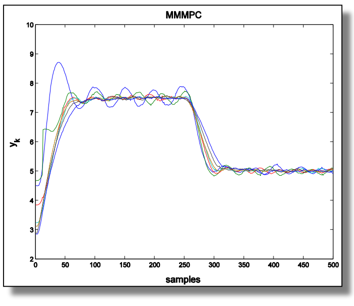
Comparisons

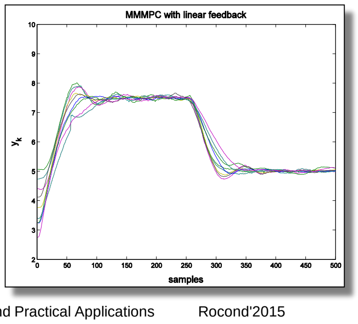
Different positions of the inlet throttle from 20º to 100º

MPC



MMMPC





MMMPC with linear feedback

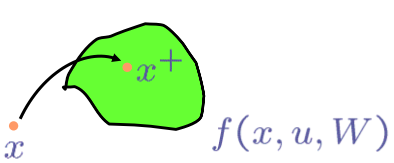
Outline

Modelling bounded uncertainties: Difference inclussion

•Model to confine the sucessor state into a set

•The **function *f(.,.,.)* and the set *W*** provide a **difference inclussion** for the system if for any pair ***(x,u),*** there is a ***w*** in ***W*** such that

$$x^+ \in \{\ f(x,u,w)\ :\ w \in W\ \} = f(x,u,W)$$



Difference Inclussions

- Example:
  - Suppose that we obtain a nominal linear model around an operation point : $x^+ \approx Ax + Bu$
  - Suppose that we are able to bound the discrepancy between the nominal model and the actual behaviour of the system:

$$\|x^+ - Ax - Bu\| \leq \rho$$

  - Thus we obtain the following difference inclussion:

$$x^+ \in \{ Ax + Bu + w \ : \ \|w\| \leq \rho \}$$

  - This can be rewritten using the Minkowski sum notation:

$$x^+ \in Ax \oplus Bu \oplus W, \text{ where } W = \{ w \ : \ \|w\| \leq \rho \}$$

Consistent state set

- The function $g(\cdot,\cdot,\cdot)$ and the set V provide a inclusion of the output of the system if

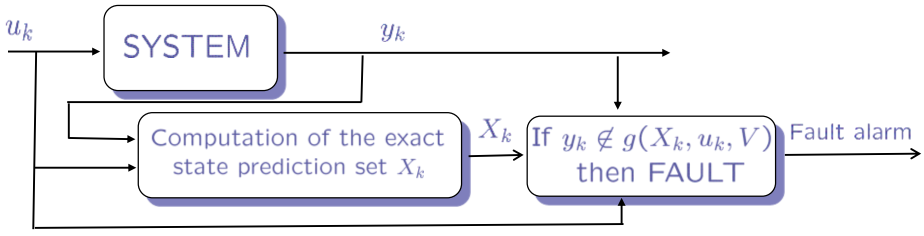
$$y \in \{ g(x,u,v) \ : \ v \in V \} = g(x,u,V)$$

- Given u_k and the measurement y_k, the **consistent state set** $\Gamma(y_k,u_k)$ is defined as the set

$$\Gamma(y,u) = \{ x \ : \ y \in g(x,u,V) \}$$

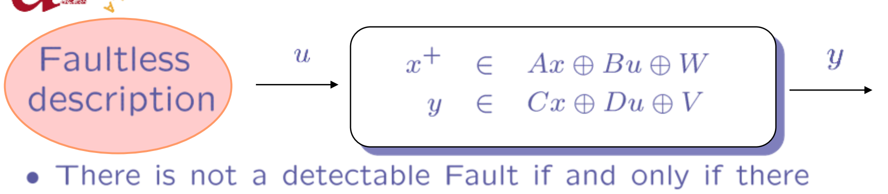
- In a faultless situation, the state at sample time k is contained into $\Gamma(y_k,u_k)$.

Determination of the compatible output set



- However, from a practical point of view, this scheme is not implementable because in most situations it is very difficult to obtain the exact uncertain sets.

A simplified case



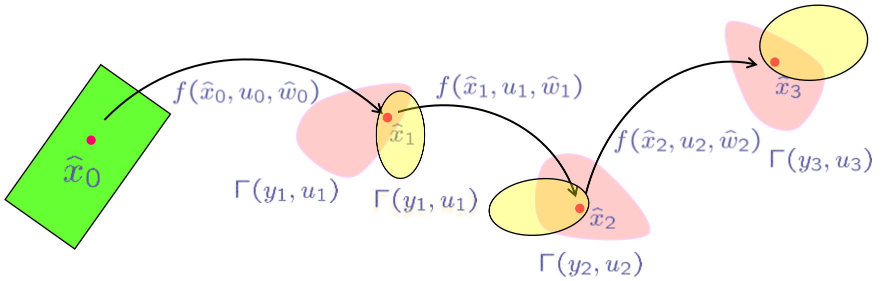
- There is not a detectable Fault if and only if there exists sequences

$\hat{x}_0, \ldots, \hat{x}_k, \hat{w}_0 \in W, \ldots, \hat{w}_{k-1} \in W$ and $\hat{v}_1 \in V, \ldots, \hat{v}_k \in V$

such that
$$\left\{ \begin{array}{rcl} \hat{x}_0 & \in & X_0 \\ \hat{x}_{i+1} & = & A\hat{x}_i + Bu_i + \hat{w}_i, \; i = 0, \ldots, k-1 \\ y_i & = & C\hat{x}_i + Du_i + \hat{v}_i, \; i = 0, \ldots, k \end{array} \right.$$

- If W and V are convex sets, this feasibility problem can be solved in an affordable time provided k is not too large.

Non Detectability



Consistent state set for model 1

Consistent state set for model 2

Non detectable faults: Multimodel MPC

Suppose a series of M model compatible with the last set of measurements.

$$x_j(t+1)=A_j(t)\, x_j(t) + B_j\, u(t) + e_j(t)$$

$$y(t) = C_j\, x_j(t) + v_j(t)$$

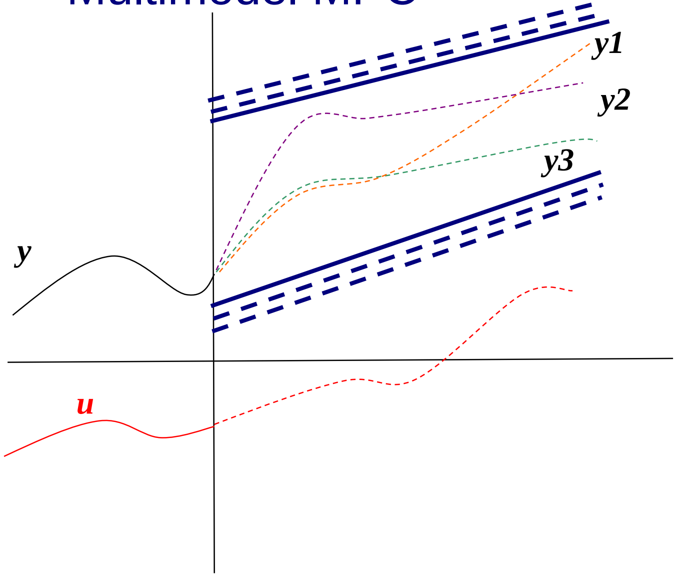
When a control sequence U is applied, the prediction equation for each active model (i.e. $j=1,2, \ldots M$)

$$Y_j = F_j\, x_j(t) + Gu_j\, U + Gw_j\, W_j$$

Each model is constrained (incuding stability and/or robustness constraints) by

$$R_j\, U \leq b_j + d_j\, x_j(t) + f_j\, W_j$$

Multimodel MPC



Multimodel MPC

$$\min_{U} \quad J^*(U, x_1(t), x_2(t), \ldots x_M(t))$$

s.t.

$$R_j U \leq b_j + d_j x_j(t) + f_j W_j \qquad j = 1, \ldots, M$$

$$J^* = \max_{W_1, W_2, \ldots W_M} J(U, x_1(t), x_2(t), \ldots x_M(t), W_1, W_2, \ldots W_M)$$

$$J^* = \quad E[J(U, x_1(t), x_2(t), \ldots x_M(t), W_1, W_2, \ldots W_M)]$$

QP problem !!!

Hypothesis on future faults

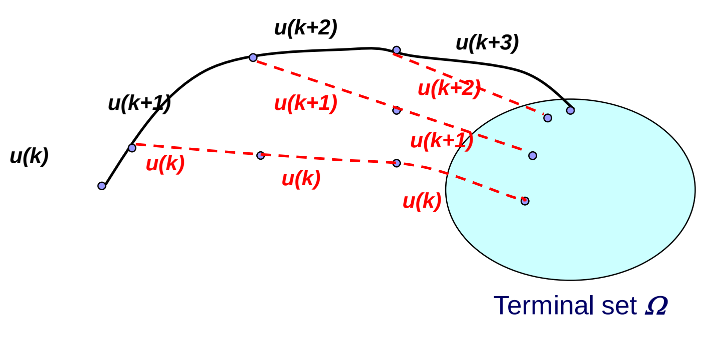
- Example: actuator jamming

Define: $U_k = [u(t), u(t+1), \ldots u(t+k-1), 0, \ldots 0]$

$$\min_{U} \quad J^*(U, x_1(t), x_2(t), \ldots x_M(t))$$

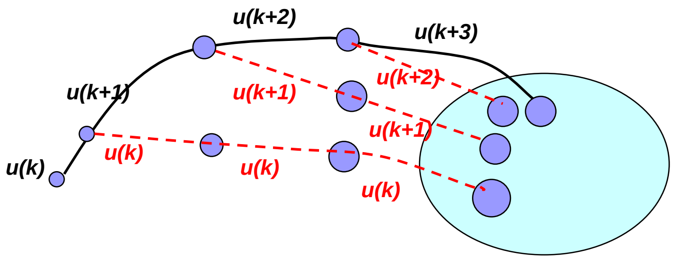
$$\text{s.t.} \quad R_j U_k \leq b_j + d_j x_j(t) + f_j W_j \qquad j = 1, \ldots, M, \;\; k = 1, \ldots, N$$

Hypothesis on future faults



Hypothesis on future faults

Robust MPC scenario



Conclusions

1. Nominal stable MPC shown to be ***input to state stable***
2. Although there are ***robust (or stable) MPC design techniques*** developed in the academia, these are ***not used in industry.***
3. Number of ***difficulties***: modelling uncertainties, determining invariant regions, computing reach sets, solving optimization problem…
4. ***Efforts needed to simplify robust design techniques***
   - Simpler models ? >> bigger uncertainties bound.
   - Heuristics.
   - Can stability be guarantied 100% ?
   - Probabilistic approaches ?

Thanks

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